The Principal Principle: Optimal Modification of Distressed Home Loans

Why Lenders should Forgive, not Forsake Mortgages

Sanjiv R. Das [a,*],

[a] *Santa Clara University, Leavey School of Business, 500 El Camino Real, Santa Clara, California, 95053, USA*

Abstract

Lenders will often restructure a loan rather than foreclose on a property because it is less value-destroying. A loan modification primarily entails a change in the loan rate, principal balance and/or remaining time to maturity; other loan features may be modified too. We analyze optimal loan modification schemes in a stochastic home price environment. Lenders maximize their loan values by minimizing the value of the borrower's option to default on the loan. Depending on the level of interest rates and home price volatility, different prescriptions apply. We argue that, controlling for the borrower's ability to pay, loan modifications via rate reductions and maturity extensions are suboptimal, leading to dissipation in loan value to the lender, and resulting in a high probability of re-default by homeowners even after modification of their loans. In contrast, loan write-downs (the Principal Principle), not a favored recipe, and sometimes prohibited by covenants, are mostly optimal. A recent innovation, the shared appreciation mortgage, enhances the ability to pay, mitigates adverse selection against lenders, and reduces the present value of expected deadweight foreclosure costs.

* Corresponding author.
Email address: srdas@scu.edu (Sanjiv R. Das).

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1 Introduction

The current housing crisis has resulted in several homeowners defaulting on their mortgages. Foreclosure filings numbered 321,480 during May 2009 (approximately 1 of every 400 homes), up 18% from the previous year.[2] When dealing with failing borrowers, lenders need to decide between modifying the loan or foreclosing. Modification results in a lowering of the present value of the loan. Foreclosing incurs deadweight costs that reduce the recovery value of the home. This paper develops a framework for reducing foreclosure losses by loan modification, and how best to implement it. Foote, Gerardi, Goette and Willen (2009) estimate that loan modification versus premature foreclosure will save $180 billion, or 1% of GDP a year.

An optimal loan modification must be cognizant of the borrower's ability to pay *and* willingness to pay. The optimal strategies of the lender (the bank/ servicer[3]) and the borrower (the homeowner) depend on the value of an American (more appropriately, Bermudan) put option, i.e., the option the borrower has to put the house back to the lender—see Kau and Keenan (1999); Deng, Quigley, and van Order (2000); and Ambrose, Capone and Deng (2001). This option determines the borrower's willingness to pay. Finding an optimal loan modification scheme requires an analysis of the game between the borrower and lender, where the lender chooses the modification scheme to maximize loan value by minimizing the value of the option to default held by the borrower, and the borrower chooses an optimal default strategy given loan parameters chosen by the lender. Simply put, the optimal strategy is analogous to the borrower's strategic exercise of an American-style put option to default on the home loan. Guiso, Sapienza and Zingales (2009) find that 26% of defaults are strategic in nature. Cohen-Cole and Morse (2009) provide evidence that foreclosure in the presence of ability to pay may arise from a precautionary liquidity motive, i.e., faced with a choice between paying a home loan and a credit card, liquidity concerns drive borrowers to pay the credit card first. The strategic default by homeowners has parallels in the corporate debt literature—see Anderson and Sundaresan (1996).

Current loan modification schemes used in practice focus on adjusting the loan terms to make the loan affordable on a monthly payment basis, but pay less attention to the borrower's default incentives in the game between the lender and borrower. Ignoring this important incentive effect makes the loan modification suboptimal, leading to dissipation in loan value to the lender, and an

[2] USA Today, 6/11/2009.

[3] Whereas loan modifications are undertaken by the loan servicer in most cases, and sometimes by investors who buy distressed loans from banks, we will use the term "lender" throughout the paper as the nomenclature of the entity that is tasked with making loan modifications.

extremely high probability of re-default by homeowners even after modification of their loans. In particular, we show that it is not optimal to implement rate-reductions or maturity extensions, but the correct prescription is to write down loan balances, which lenders are reluctant to do. Meadows (2009) and the FDIC (2009) Loan Modification Plan focus on these issues as well, but stop short of providing an option-based analysis. Given the severe costs of foreclosure—many articles, for example see Ambrose and Capone (1996) for an analysis of foreclosure and modification alternatives—suggest that loan modification to reset the ability and willingness to pay of the borrower is more often the socially better outcome. In contrast to this prior work, this paper focuses specifically on the willingness to pay (strategic default) across various modifications, *after* equalizing them on the ability to pay. Hence, we provide insights into the optimal modification of loans to maximize the economic value of the loan to the lender after accounting fully for the willingness to pay of the borrower.

The intuition for the optimal loan modification is simple. The first step lies in correctly determining the level of monthly service payments on the loan that are affordable to the distressed borrower. This calibrates the loan to the borrower's *ability* to pay. The second step involves the borrower's *willingness* to pay and foreclosure costs. Since there are many loan configurations that lead to the same monthly payment level, we show how to select the one that is most favorable in maximizing the value of the loan to the lender. Our analysis involves a value decomposition of the loan into (a) a risk-free component, (b) a default put option, and (c) a component for expected deadweight foreclosure costs. The optimal loan configuration maximizes lender net value aggregated across all three components.

The main results of the paper are as follows. In Section 2 we present the mathematical framework in which we analyze the optimal loan modification problem in the presence of stochastic home prices. The model is solved numerically using a simple lattice framework. Section 3 shows that rate reductions are suboptimal. Maturity extensions are also usually suboptimal especially when home price volatility is high. Maturity extensions may be better in a few cases, when the deadweight foreclosure costs are low, and loan rates are high. Most important, writing down principal on the loan is better than reducing rates and extending maturity. Section 4 examines whether these results are qualitatively sensitive to the deadweight costs of foreclosure and finds that they are not; of course, quantitatively, deadweight foreclosure costs will reduce net home value for the lender. Examination of the default barriers for the borrower show that strategic default occurs only when there is significant negative equity, a proposition that has recent empirical backing. The negative equity problem is further exacerbated by the phenomenon of cash-out refinancings, where equity in the home is reduced by all homeowners simultaneously in an environment of falling interest rates and rising house prices, leading also to systemic increases in home loan default risk—see Mian and Sufi (2009); Khandani, Lo and Merton (2009). The loan-to-value (LTV) level at which a homeowner defaults is usually lower when home price volatility is low than when it is high. As loans age, the threshold level of negative equity needed to trigger default becomes smaller. Section 4 also presents an empirical analysis of modified loans and finds that loan re-default rates decline with suitable modifications. The section also analyzes a recent innovation—the shared appreciation mortgage (SAM)—and shows that such loans improve the borrower's ability to pay, mitigate deadweight foreclosure costs, and adverse selection in the loan modification market.

In Section 5, we compare our theoretical prescriptions to what is being done in practice. It appears that practice contradicts theory and there is evidence of high re-default rates after loan modifications. However, hedge funds and other investors that buy pools of distressed mortgages appear to be making optimal modifications by writing down loan balances. Section 6 offers discussion and concluding comments.

2 The Problem

We define troubled loans at a given time t as a home loan having negative equity, i.e., $E_t < 0$. The loan balance on the home is denoted L_t. The value of the home is driven by a stochastic process V_t with dynamics given by a geometric Brownian motion, i.e.,

$$dV_t = \mu V_t \, dt + \sigma V_t \, dZ_t \tag{1}$$

where μ is the mean growth rate of the home's value with variability determined by the volatility coefficient σ. The randomness comes from a standard Brownian motion increment dZ. Kelly (2006) points to the considerable evidence showing that the level of homeowner's equity is a major determinant of default. Changes in equity, known as contemporaneous equity (gains and losses), are an even more critical determinant of foreclosure than initial equity. Therefore, the modeling of home equity movements is essential to the analysis of loan modifications.

For a given loan balance L_0 with time T remaining on the loan, A is defined as the total flat payment per year including principal and interest. This is a function of the loan balance L_0 and the interest rate on the loan, which we denote r_L per annum. Inferring A is easy because the annuity stream must equal the present value of the loan, i.e., L_0. The annuity equation resulting in the value of the loan in continuous time is

$$L_0 = A \int_0^T e^{-r_L t}\, dt = A\left(\frac{1 - e^{-r_L T}}{r_L}\right) \tag{2}$$

Re-arranging this equation we get the flat payment rate per annum

$$A = \frac{r_L L_0}{1 - e^{-r_L T}} = \frac{r_L L_t}{1 - e^{-r_L (T-t)}} \tag{3}$$

As the equation states, the same flat payment applies irrespective of when the annuity is calculated.

In discrete time with (usually monthly) payments, we get the well known equation for periodic flat payments, analogous to that in equation (3):

$$A = m \times \frac{i L_0}{1 - (1+i)^{-N}} \tag{4}$$

where A is the annual total of payments, $i = r_L/m$ is the periodic interest rate, and the number of remaining periods (months) is $N = mT$, where m is the number of periods in a year ($m = 12$ for monthly payments, which is standard in the US).

The corresponding law of motion for the principal balance at any period n is

$$L_{n+1} = L_n \left[(1+i) - \frac{i}{1 - (1+i)^{-(N-n)}}\right] \tag{5}$$

Note that $0 < L_{n+1} < L_n < L_0$ for all n. The quantity in brackets on the right-hand side of the preceding equation is the proportion of principal balance remaining after period $(n+1)$.

The loan-to-value ratio R_t is given naturally as L_t/V_t. We also note that $E_t = V_t - L_t$ and we may write $R_t = 1 - E_t/V_t$.

The homeowner has an incentive to default on the loan whenever $R_t > 1$ which is the same as when $E_t < 0$. Note that if $E_t > 0$, it is always better for the homeowner to sell the house instead and pay off the loan, thereby retaining the residual equity. The Wall Street Journal reported on August 5, 2009 that 24% of owner-occupied single-family homes had mortgage debt exceeding home value, i.e., negative equity. The situation was even more critical in states like Nevada (40% of homes with negative equity), Arizona (37%), and California (33%).

Effectively, the homeowner has an American put option on the home value V_t at a strike price of L_t. The strike price changes each period because the loan balance changes as per the dynamics in equation (5). Because this put option has time value, it is not always optimal to exercise the put when it is just in-the-money, i.e., when $V_t = L_t$, but it might be worth waiting to exercise when it is more valuable. Also, there are costs to exercising the put on one's home such as finding a new home (which we denote as "relocation costs" K_R), and these make this put option different from standard puts on stocks and bonds. Hence, the homeowner will exercise his option to renege on the loan when it is sufficiently underwater, i.e., $R_t \gg 1$ so as to be optimal after relocation costs. Foote, Gerardi, Goette and Willen (2009) provide evidence that borrowers foreclose when it is in their economic interest to do so, signifying that they appear to optimally exercise their strategic option to default.

Explicitly, the put option at the current time ($t = 0$ without loss of generality) may be written as

$$P_0 \equiv P(V_0, L_0, T, r, r_L, \sigma, K_R) \tag{6}$$

where V_0, L_0 are the current values of the home and loan balance, T is the remaining maturity on the loan, and r is the discount rate equal to the risk free rate.

Given that the strike price L_t of this default put option changes each period, and the option is American, we price the option on a lattice using the following recursive equation:

$$P_t = \max[E(P_{t+h})e^{-rh}, L_t - V_t - K_R + A/m], \quad \forall t \leq T \tag{7}$$

Equation (7) is implemented by backward recursion to determine the value of the American option, accounting for the optimal default policy. At every mortgage payment date, the homeowner will choose to default if the value of immediately exercising this default put ($L_t - V_t - K_R + A/m$) is greater than the current expected value of the put if it is not exercised, i.e., $E(P_{t+h})e^{-rh}$ (also known as the "continuation value"). The value on exercise is the amount of negative equity ($L_t - V_t$) minus the relocation costs (K_R) plus the gains from not making one payment (A/m). Whether the last term A/m is taken into account in assessing the value of immediate exercise of the default put depends on whether we assume that default occurs at the beginning of the payment period or at the end of the period. If default occurs at the beginning of the period, the term should be set to zero. We assume this to be the case in all the analyses in this paper; we have checked that the results remain the same qualitatively, and change in no material way quantitatively.

Even though the house is a fairly non-tradable asset from the point of view of the homeowner, especially in a poor housing market, the put option to return the asset to the lender is a liquid option and may be exercised at any time. Hence, risk-neutral pricing is applicable, and the risk free rate may be used for pricing as the option to put the home is not impacted by illiquidity or non-tradability for the homeowner. Moreover, from the point of view of the lender (a large bank, say), the pool of home assets may be treated as tradable. In a market where foreclosed assets become less tradable, the value of the option and the exercise policy of the homeowner do not change, but the cost to the lender does change, and we will consider this extension later in the paper.

The lender's valuation of the loan (denoted B_t) incorporating the default option P_t is made up of two components: (a) the present value of a credit-risk-free annuity stream (denoted H_t) minus (b) a default option (put) held by the homeowner:

$$B_t = H_t - P_t, \qquad 0 \leq t \leq T \tag{8}$$

As stated earlier, the fixed interest rate on the loan is denoted r_L, and we denote the current level of interest rates in the market as r_C (the mortgage rate). If these two rates are equal then $H_t = L_t$.

In discrete time, if $r_L \neq r_C$ then we have that after n periods

$$H_n = \frac{A}{m} \times \left[\frac{1 - \frac{1}{(1+i)^{(N-n)}}}{i} \right], \quad i = r_C/m \tag{9}$$

which is the just the present value of the stream of payments A/m discounted at the periodic rate i, where, following from equation (4),

$$A/m = \frac{(r_L/m)L_n}{1 - (1 + r_L/m)^{-(N-n)}} \tag{10}$$

and m is the payment frequency per year ($m = 12$ if monthly).

By treating the market mortgage rate r_C as non-stochastic, we abstract away from the fact that as interest rates r_C change, the value of L_t will also change. However, we make this simplifying assumption to focus on the main issue, i.e., the default put, and how the lender will restructure it to maximize the value of the loan.

2.1 Modification

In the event of the homeowner defaulting on the loan, the bank has the option to foreclose or to modify the loan. A loan modification at time t involves changing the remaining maturity of the loan $(T - t)$, or the principal balance L_t, or the loan rate r_L. These modifications translate into a lower flat payment A that is affordable to the borrower. There may be many level sets of A that are obtained by various combinations of $\{T, L_t, r_L\}$. These may be thought of as indifference curves from the point of view of the borrower. Among these level sets, the lender will want to choose that set which maximizes the value of B_t, i.e., the one that minimizes the value of the default option P_t and maximizes the value of H_t (see equation 8).

The distressed borrower has a limited ability to make periodic loan payments. Assume that this is specified by a maximum level of periodic payment A_{max}. Any loan modification scheme must be such that it induces the borrower to keep making payments, and in this setting, we need to choose $\{T, L_t, r_L\}$ such that

$$A \leq A_{max} \tag{11}$$

This is the feasibility region for the loan modification. We also note that A_{max} may be set by a regulatory agency such as the FDIC. It may be expressed as a function of the borrower's income, known as the housing-to-income (HTI) ratio. The FDIC (2009) plan suggests an HTI of 31-38%.

We note from equation (10) that the three choice variables—maturity T (or number of periods N), loan balance L, and loan rate r_L—are also the determinants of the flat periodic payment A. Hence, while modifying a loan, if we hold fixed a given level of periodic payment A, then these three variables are not all free, as we have lost one degree of freedom in requiring that the periodic payment A be less than or equal to A_{max}.

We are now ready to state the optimization problem of the bank (lender). Assume a modification at time $t = 0$, without loss of generality. Given that the borrower cannot make his payments, i.e., that currently $A > A_{max}$, the bank will solve the following problem.

$$\max_{\{\mathrm{T}, \mathrm{r_L}, \mathrm{L_0}\}} \quad B_0(T, r_L, L_0) \tag{12}$$

subject to

$$B_0 = H_0(L_0, T, r_L, r_C) - P_0(L_0, T; V_0, r, r_L, \sigma, K_R) \tag{13}$$

$$A = \frac{r_L L_0}{1 - e^{-r_L T}} \leq A_{max} \tag{14}$$

More descriptively, taking $\{V_0, r, \sigma, K_R, r_C\}$ as given, find the best r_L, L_0 and T to maximize the value of the loan to the lender, subject to the constraint that the modified loan is affordable to the borrower ($A \leq A_{max}$). We also require that the modified loan be worth more than the value achieved from foreclosure, i.e., we want $B_0 > V_0$. Else, the lender would prefer to foreclose than to modify the loan. Therefore, if there is no feasible solution to this problem, then we conclude that it is better to foreclose.

Equation (13) shows that B_0 has two terms, a risk free loan H_0 and a put option P_0. A similar decomposition appears in the paper by Merton (1974) but that model deals with simpler zero-coupon debt. Note that L_0 is similar to the strike price of the default put option. If we reduce loan balance L_0, the first term, the annuity, declines in value but the second term, the put option, also declines in value and hence it may be value increasing overall to write down principal. Increasing T raises the value of the put and the loan value to the lender declines monotonically as T is increased. Reducing the interest rate r_L on the loan will reduce the flat payment rate A, thereby loosening the constraint and allowing better modulation of T and L_0. But, to look at these univariate changes in isolation is misleading as we will see. Keep in mind that if we keep $A = A_{max}$ fixed, then changes in any of the terms of the loan impact the other terms of the loan and hence the net impact on the value of the loan is not intuitively obvious.

We undertake a comparative analysis to determine which parameter has the most "bite" in the loan modification process, and in which direction it is optimal to move the parameter when we modify the loan. Since the option is American, we will need to use a numerical scheme. The simplest approach for pricing the option is to use the binomial tree model of Cox, Ross and Rubinstein (1979). The risk-neutral home value process V_t is modeled using a recombining tree. The default put is priced on this tree assuming a strike price L_t at each time t, because the strike price L_t changes deterministically in time as in equation (5). The default put is priced using backward recursion as is standard in these tree models, and early exercise is undertaken when the current value of the put P_t is less than the immediate exercise value of the default option, i.e., $L_t - V_t - K_R$, where we recall that K_R is the relocation cost. If we extend this model to stochastic interest rates, then L_t will change stochastically and the tree will need an additional dimension; however, as mentioned earlier, we abstract away from this complication. Note, however, that Deng, Quigley, and van Order (2000) do show that the commingling of the default option (dependent on V) and the refinancing option (dependent on r_C, r_L) is an important feature to be modeled.

We assume that the reader is familiar with Cox-Ross-Rubinstein binomial tree models and their original paper may be referenced for details on building option-pricing trees. In this paper we use a variation of the Cox-Ross-Rubinstein model developed by Jarrow and Rudd (1983). [4]

In the next section, we use our tree model to analyze the problem to uncover optimal loan modification schemes.

[4] Given these lattice models are well-known we do not provide a detailed exposition here.

3 Analysis

The initial driver of the loan modification decision is the debt service level that the borrower can maintain per year. This amount is denoted A_{max} as stated earlier. The amount paid per payment period is A_{max}/m, where m is the frequency of payments per year (we assume $m = 12$ in the examples here). If there are N remaining payment periods, and we set a loan rate r_L, then given A_{max} we can solve for the implied loan balance by re-arranging equation (10):

$$L_0 = \frac{A_{max}}{m}\left[\frac{1-(1+r_L/m)^{-N}}{r_L/m}\right] \tag{15}$$

This equation gives the loan balance L_0 that is needed with loan rate r_L and maturity T to provide exactly A_{max} in loan service per year. If we construct a 3D plot of L_0 against $\{r_L, T\}$, we obtain an "iso-service" loan level graph, i.e., a plot of L_0 against r_L and T where the debt service requirements remain constant. For an example, see the left side plot in Figure 1.

To fix ideas, assume there is a home with a current loan balance of \$300,000 but that the value of the home has fallen to $V_0 = \$250,000$, i.e., there is negative equity. The current remaining maturity on the loan is $T = 25$ years, the loan rate is $r_L = 6\%$, the current market level of the mortgage rate is $r_C = 6\%$. Assuming a monthly flat payment comprising principal and interest, equation (4) may be used to calculate the monthly payment. Given the values above, this amounts to \$1,932.90 (an annual amount of \$23,194.80). Suppose the lender determines that the borrower has no ability to pay unless the monthly service amount is lowered to \$1,666.67 (annually \$20,000) then the terms of the loan $\{r_L, T, L_0\}$ will need to be modified accordingly. The best result for the lender is obtained by implementing the optimization program in equation (12), to find the highest loan value B_0 subject to remaining on the iso-service surface where the monthly service level is constant and equal to \$1,666.67.

Figure 1 shows the iso-service loan surface on the left-side plot and the loan value surface on the right-side plot. We can see that as the interest rate (r_L) rises, keeping maturity T constant, to remain on the iso-service surface for annual payments totaling $A_{max} = \$20,000 (= \$1,666.67 \times 12)$, the loan balance L_0 must be reduced, i.e., the lender will write down the value of the loan. On the other hand, keeping interest rates fixed, if the lender increases the maturity of the loan, the loan balance will increase in order to remain on the iso-service surface. The latter situation is what is normally done by a bank when modifying a home loan, i.e., the maturity is extended and principal is increased, usually by the number of unpaid loan installments.

In Figure 1, the right-side plot shows the value of the loan computed using equation (13) subject to the constraint in equation (14). The main result

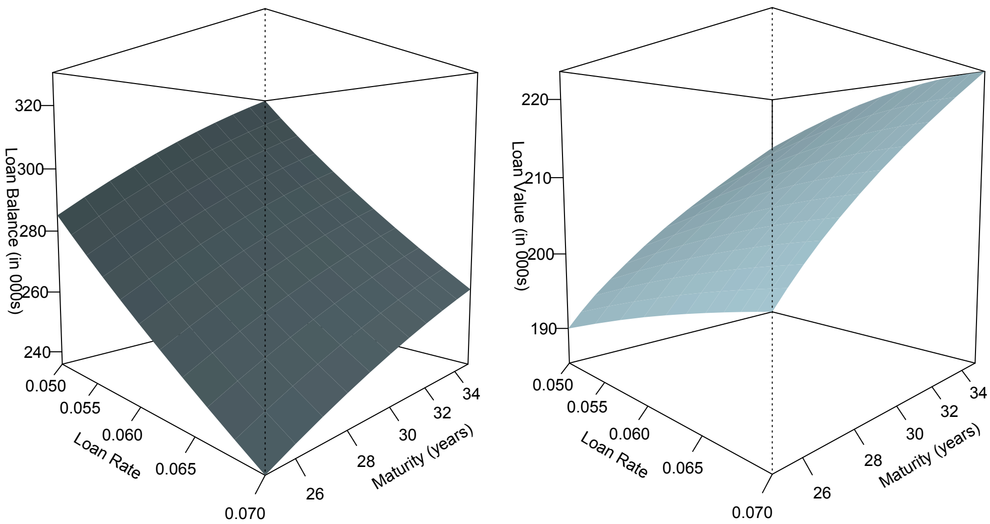

Fig. 1. Loan balance and loan value for equal loan service. The plots are shown as the loan rate (r_L) and maturity (T) vary, keeping the debt service fixed at $A_{max} = \$20,000$ per year ($1,666.67 a month). The plot on the left shows what the loan balance L_0 needs to be to constrain debt service at this level computed using equation(15). The plot on the right shows that value of the loan for the given levels of $\{r_L, T, L_0\}$. The inputs used are: $V_0 = 250,000$, $\sigma = 0.3$, $r_f = 0.03$, $K_R = 0$, $n = 300$, $r_C = 0.06$, $m = 12$.

in the figure is that the higher the loan rate, the greater the value of the loan. Modifying the loan by increasing its maturity is usually value dissipating (reduces loan value B_t) for the lender, except when the loan rate is set to be the current market mortgage rate. From the figure, we see that if the loan rate is low (say 5% compared to the market mortgage rate of 6%), then loan modification by increasing the maturity of the loan reduces the value of the loan to the lender. For loan rates above 5.5%, we see that the loan value rises when maturity is extended.

The intuition for why rates should be raised comes from an examination of the two components of B_0 in equation (13). The first component H_0, the present value of a default free loan, rises as the loan rate is increased. The second component is the default put P_0 held by the borrower. Notice from the left hand side plot in Figure 1 that when interest rates are increased, the loan balance on the iso-service surface declines. Since the loan balance is effectively the strike price of the default put held by the borrower, reducing the strike makes the default put less valuable. Therefore, staying on the iso-service surface while raising the loan rate results in H_0 rising and P_0 declining, thereby raising the loan value B_0. Doing the opposite, i.e., implementing a rate reduction, clearly loses value.

Of course, increasing the loan rate above the market's mortgage rate for the same credit quality borrower is infeasible, as the borrower would simply refinance, unless such refinance were not available on account of a market break-

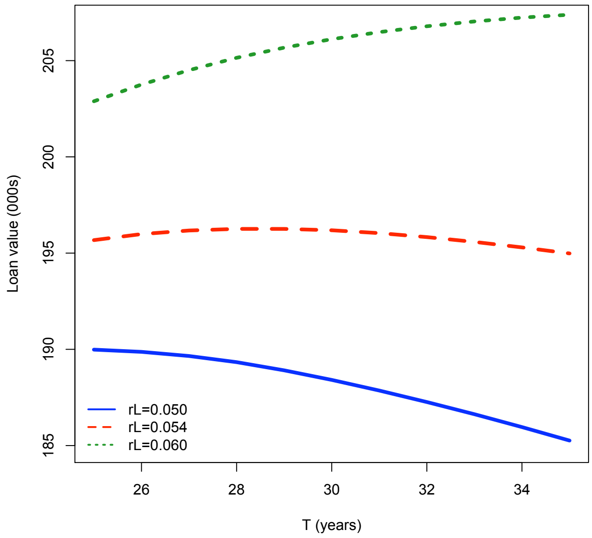

Fig. 2. Loan values for different maturities as the loan rate is varied. When the loan rate is $r_L = 0.05$, then if maturity is extended, the loan value to the lender B_0 declines. If the loan rate is somewhat higher, i.e., $r_L = 0.054$, then there is an optimal maturity (roughly 28.5 years) at which the loan value to the lender is maximized. When the loan rate is even higher ($r_L = 0.06$), the loan value increases with maturity. This is because the increased maturity results in a lower write down of the principal balance. The parameters of the plot are the same as in Figure 1. These plots are just slices taken from the right side graph in Figure 1.

down. Hence, it seems clear that raising the loan rate up to the current market level is one prescription, i.e., set $r_L = r_C$. Having done that, the lender should then select the suitable maturity and lower the loan balance to the extent feasible, making sure to remain on the same iso-service surface.

We note that at intermediate loan rates there may be an optimal maturity. This is shown in Figure 2. At a loan rate of $r_L = 0.054$, with a market mortgage rate of $r_C = 0.06$, raising maturity beyond 25 years does raise the overall value of the loan to the lender, but beyond a point (approximately 28.5 years in this example), the increased maturity makes the put option more valuable and then starts degrading the value of the loan.

To summarize: We see in Figure 2 that when rates are low, we should lower loan maturities, and when rates are high, we may raise loan maturities. At intermediate levels of the loan rate, there will be an optimal maturity for the loan.

In all these cases, we also note that the maximized value of the loan B_0^* after modification will be compared to the value of the loan V_0. If $B_0^* < V_0$, then the optimal decision of the lender is to foreclose.

How do these results change when home price risk is low versus when it is high? We used an annualized home price volatility of $\sigma = 0.30$ in the preceding example. We now examine the cases when $\sigma = \{0.15, 0.45\}$. The results for these cases are presented in Figure 3. In the left-side plot we see the value of the loan when home price volatility is $\sigma = 0.15$ per annum. Here, the results are similar to those obtained in the earlier case when $\sigma = 0.30$. That is, when the loan rate is low, reducing loan maturity is value maximizing, and when rates are higher, increasing maturity is better for the lender.

The right-side plot of Figure 3 shows that the result changes when home price volatility is very high. Loan modifications that increase loan maturity may now reduce the value of the loan to the lender. The intuition is simple – at high volatility, the increase in maturity of the loan makes the option to default P_0 very valuable and swamps the effect of the increasing value of the risk free component H_0 of the loan. The prescription here then is that when house values are very volatile, the lender should shorten the loan maturity, raise loan rates r_L to the market mortgage rate r_C, and write down the loan balance to the amount described by equation (15). We note that when $r_L = r_C$, the loan value is mostly higher at low maturities, and the loan is maximized by extending the maturity only slightly to 26 years. Once again, there is an optimal maturity that maximizes loan value.

We summarize these results in Table 1. The table shows the various conditions under which loan modifications may be made, and the prescriptions that follow from the preceding analysis.

The first modification is made under the condition that the loan rate is substantially below the current mortgage rate. Then, irrespective of home price volatility, the prescription of the model is to keep the maturity of the loan low. This of course has the effect of necessarily requiring a write down of some amount of the loan balance (L_0), thereby reducing the risk free component of the loan value (H_0), but also reducing the value of the default option (P_0) held by the borrower. The net effect is to increase the loan value B_0 to the lender.

The second modification sets the loan rate equal to the current mortgage rate (i.e., $r_L = r_C$). When house price volatility is low, the model prescription is to extend loan maturity. Keeping debt service constant, this will increase the loan balance (this would depend on the contracting environment). What this does is raise the risk free component of the loan value (H_0), and also increases the value of the default option (P_0) held by the borrower. The net effect however is to increase the loan value B_0 to the lender.

The third modification sets the loan rate equal to the current mortgage rate (i.e., $r_L = r_C$) but in an environment of high house price volatility. Here the model prescription is to reduce loan maturity. Keeping debt service constant,

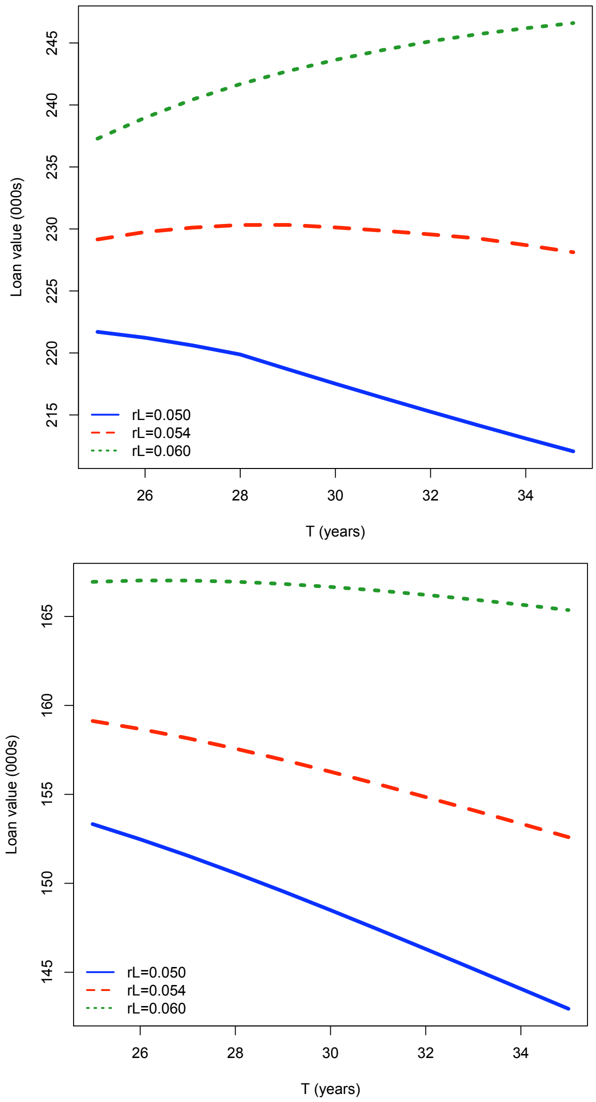

Fig. 3. Loan values for different maturities as the loan rate is varied. When the loan rate is $r_L = 0.05$, then if maturity is extended, the loan value to the lender B_0 declines. If the loan rate is somewhat higher, i.e., $r_L = 0.054$, then there is an optimal maturity (roughly 28.5 years) at which the loan value to the lender is maximized. When the loan rate is even higher ($r_L = 0.06$), the loan value increases with maturity. This is because the increased maturity results in a lower write down of the principal balance. The parameters of the plot are the same as in Figure 1 except that the value of home value volatility is taken to be $\sigma = 0.15$ (left plot) and $\sigma = 0.45$ (right plot), respectively.

this will reduce the loan balance L_0 (i.e., a write-down). What this does is lower the risk free component of the loan value (H_0), and also lowers the value of the default option (P_0) held by the borrower. The net effect is to increase the loan value B_0 to the lender.

The fourth modification is one where we keep maturity fixed and reduce the interest rate on the loan. Since the interest rate on the loan is reduced, the differential interest is capitalized into the loan balance, keeping maturity fixed.

Table 1
Loan modification prescriptions. The table shows when loan maturity should be extended and when it should be shortened. The effect on the loan balance L_0, and loan components – the risk free part H_0 and the default put P_0 are also shown.

Conditions & Prescription			Effect on:			
			Loan Balance	Risk Free	Default Put	Loan Value
Int Rate	Volatility	Maturity	after mod	Component	Option	to Lender
$r_L \ll r_c$	σ low or high	$T \downarrow$	$L_0 \downarrow$	$H_0 \downarrow$	$P_0 \downarrow$	$B_0 \uparrow$
$r_L = r_c$	σ low	$T \uparrow$	$L_0 \uparrow$	$H_0 \uparrow$	$P_0 \uparrow$	$B_0 \uparrow$
$r_L = r_c$	σ high	$T \downarrow$	$L_0 \downarrow$	$H_0 \downarrow$	$P_0 \downarrow$	$B_0 \uparrow$
$r_L \downarrow$	σ low or high	T fixed	$L_0 \uparrow$	H_0 fixed	$P_0 \uparrow$	$B_0 \downarrow$
$r_L \downarrow$	σ low or high	$T \downarrow$	L_0 fixed	$H_0 \downarrow$	$P_0 \downarrow$	$B_0 \downarrow$

This raises the loan balance. Since the flat payment is held constant as well as maturity, the risk free component of the loan remains the same. However, since the loan balance (i.e., the strike price of the default put) has ratcheted up, the value of strategic default increases, and the net effect is a reduction in loan value. The prescription that we arrive at is that a modification that holds maturity fixed and reduces interest rates is value-destroying from the lender's viewpoint.

The fifth modification reduces the loan rate and reduces maturity, keeping the loan balance unchanged. This reduces the value of the risk free component of the loan and also reduces the value of the default put. However, the net effect is a reduction in the value of the loan. Hence, lower rates in a loan modification degrade loan value on the lender's book.

Overall the intuition is that when home price volatility is high, extending loan maturity is not that beneficial to the lender, especially if rates are being lowered. This is surprising because in practice, the more intuitive extension of maturity is undertaken instead. However, in times of low home price volatility, the usual maturity extension approach is better from the lender's viewpoint. Dropping interest rates without reducing maturity will definitely lose loan value as well.

4 Further Analysis

4.1 Deadweight costs of foreclosure

So far we have assumed that homes may be resold at the current home value V_t. However, foreclosed homes may end up selling at a value $\phi V_t, 0 \leq \phi < 1$ because the housing market in a local area has become illiquid, there are costs to foreclosure, or the lender needs to get the home off its books—see Campbell, Giglio and Pathak (2008) for extensive evidence on these costs from forced sales. (These "deadweight" costs of foreclosure are akin to the deadweight costs of bankruptcy in a corporate bond setting.) These costs reduce the value B_t of the loan to the lender but do not impact the homeowner's decision to default. Does this change the basic intuitions we developed in Section 3?

We re-optimize the model using an extension of equation (13) for the value of the loan to the lender,

$$\begin{aligned} B_0 = & H_0(L_0, T, r_L, r_C) \\ & -P_0(L_0, T; V_0, r, r_L, \sigma, K_R) \\ & -D_0(\phi, P_0) \end{aligned} \tag{16}$$

where the last term $D_0(\phi, P_0)$ is the expected present value of deadweight foreclosure costs. These costs are a function of the parameter ϕ, and when the homeowner chooses to default, the lender incurs a cost equal to $(1 - \phi)V_t$. This term is calculated on the option pricing tree where at each node, if the homeowner optimally defaults, then the lender incurs the deadweight foreclosure cost. These costs are probability weighted and discounted on the tree to get the present value D_0.

Figure 4 shows the optimized loan value when deadweight costs are taken into account. The plots are undertaken for two levels of home price volatility ($\sigma = \{0.30, 0.45\}$) and for two levels of deadweight costs (20% and 40%, corresponding to $\phi = \{0.8, 0.6\}$). As deadweight costs increase (moving from left to right in the figure), the value of the optimized loans declines as is expected. The qualitative results remain the same as with the case when there are no deadweight costs and hence the cases detailed in Table 1 remain applicable. In short, the results and intuitions remain the same after consideration of deadweight costs. Quantitatively, of course, increasing deadweight costs reduce loan value to the lender.

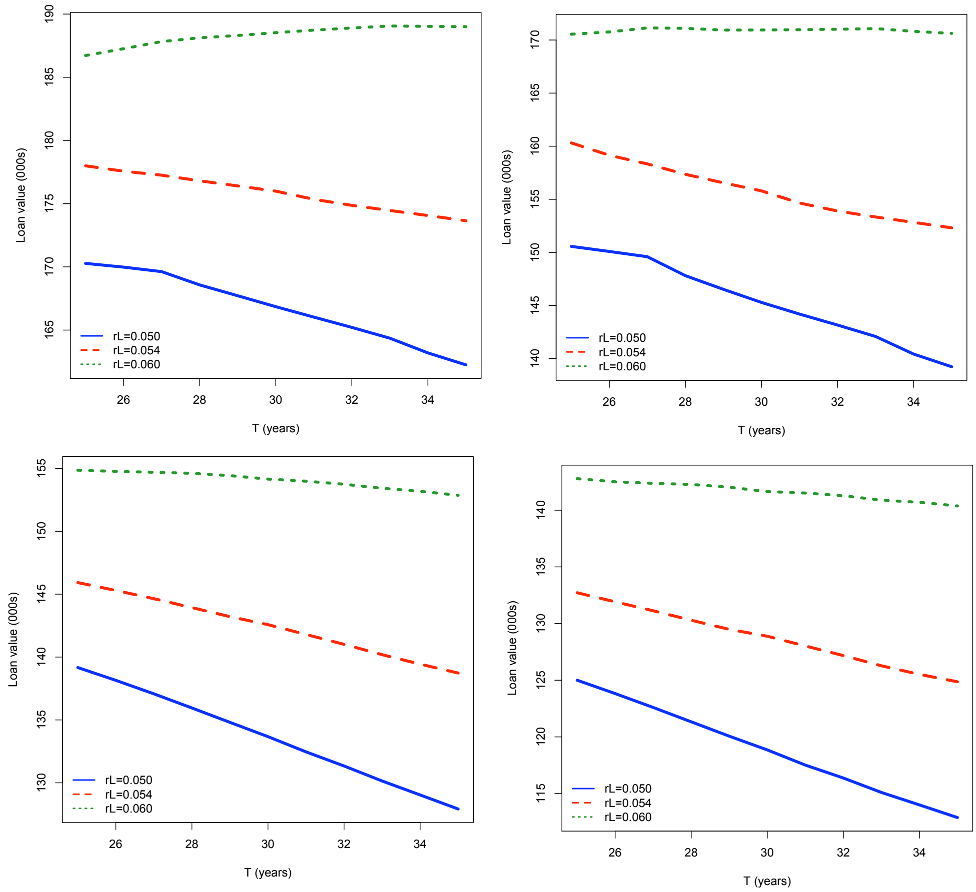

Fig. 4. Optimized loan values with non-tradability discounts. The price of the home after discounting for non-tradability is $\phi V_0, 0 \leq \phi < 1$. The left-side plots are computed for $\phi = 0.8$ and the right-side plots are for $\phi = 0.6$. The top two plots show the value of B_0^* for housing price volatility at $\sigma = 0.30$ and the bottom two plots are for $\sigma = 0.45$. The other parameters of the plot are the same as in Figure 1.

4.2 The default boundary

The shape of the default boundary offers intuition about when the default option may be exercised. Figure 5 shows that the loan value declines over time, and correspondingly the default barrier must also decline with time. Unlike the usual early exercise barrier in a vanilla American put option, where the barrier is convex and increasing, the barrier in this case is concave and decreasing. For the first half of the remaining life of the mortgages, the default barrier has a mild slope, especially for higher home value volatilities. The impact of volatility on the barriers is quite dramatic as may be seen from the two plots in Figure 5. At 15% home value volatility, homeowners will choose to default for small drops in home value, but when home value volatility is at 30%, homeowners will carry significant negative equity before defaulting, provided, of course, that they can make their payments. Guiso, Sapienza and Zingales (2009) report that no homeowner will default unless negative equity increases

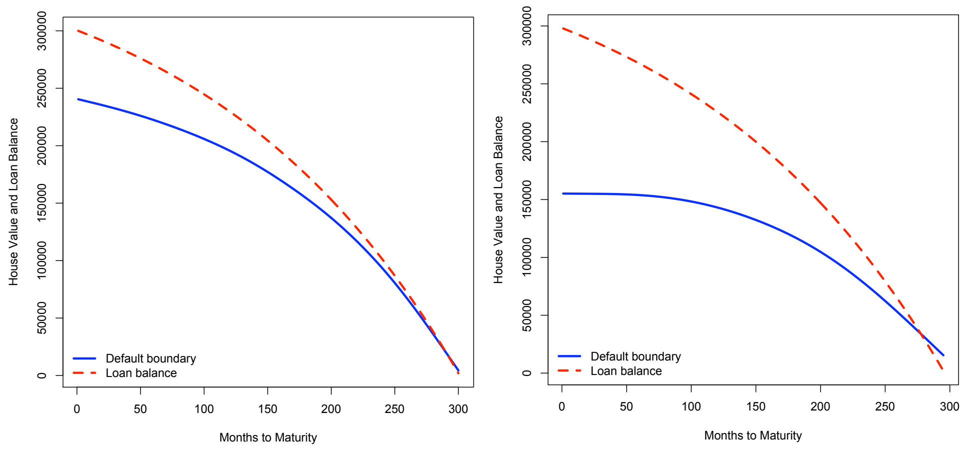

Fig. 5. Homeowner's default boundary. The plot shows the level of home value at which the homeowner chooses to default under the modified contract. The loan balance is \$300,000 and the initial home value is \$250,000. The maturity is 25 years (300 months), and home value volatility is given by $\sigma = 0.15$ for the left-side plot and $\sigma = 0.30$ for the right-side plot. The risk free rate is 3%, the mortgage rate of the loan and in the market is $r_L = r_C = 6\%$ and there are no relocation costs or deadweight costs. The plot also shows the corresponding loan balance declining over the life of the contract. Both lines on the plot have been smoothed using a splining technique to eliminate wrinkles arising from the discrete lattice calculations.

beyond 10% of home value.

4.3 Some Empirical Evidence

Whereas the pace of loan modifications has stepped up in the recent past, it is useful to ask whether these modifications have stemmed re-default rates on modified loans. For illustration, using a small sample of modified loans from a financial institution, we assessed this question using logit regressions of re-default on the amount of changes in the loan rate, principal balance, and maturity. As controls, we used the LTV of the loan, the debt ratio of the household after the loan modification, and the percentage reduction in the monthly payment on the loan. These three variables in the logit regressions are instrumental variables. The loan-to-value ratio of the home (LTV) is an instrument that captures the moneyness of the default option, and captures the willingness to default. The debt ratio of the household is an instrument for the borrower's characteristics. The percentage payment change (PPC) is the percentage amount by which the monthly payment on the loan was reduced. This is an instrument for the improvement in the ability to pay of the borrower. The sample covers two years, 2007 and 2008, and results are reported separately by year. The columns in the table stratify the analysis into five monthly payment ranges, thereby looking at the data in terms of equal ability to pay after loan modification.

We see from Table 2 that term extensions do not explain subsequent re-default rates, but that rate reductions and principal write-downs are effective in doing so. The magnitudes of the chi-square statistics show that in 2008, loan principal reductions are especially effective. Clearly the LTV of the loan also matters—the greater the LTV at modification, the higher the subsequent re-default rate. This suggests that making the default option held by the borrower less in-the-money is an effective way to contain re-default.

4.4 Shared Appreciation Mortgage (SAM)

A recent innovation in mortgage restructuring offers the borrower a reduced monthly payment in return for the lender taking a share of the upside of the home value. The lender may agree to reduce the monthly payment on a loan to bring it down to a level that is affordable to the distressed borrower; in return, the lender takes a share in the home equity, conditional on the home value increasing above the loan value. This is known as a "shared-appreciation mortgage" (SAM) or "home equity fractional interest" (HEFI) loan. The original idea for this was developed in O'Brien (2008).

We can use the framework in this paper to understand how a SAM impacts the incentives of the borrower. As discussed in Section 2 the borrower strategically defaults when the exercise value $(L - V - K_R)$ of the in-the-money option to default is greater than its continuation value. Is the borrower more or less likely to strategically default if his loan is restructured as a SAM?

Recall that a loan modification impacts both, the borrower's ability and willingness to make loan payments. Suppose the lender decides to offer a loan modification that consists of a principal write-down of W. Reducing the loan balance automatically reduces the monthly payments, thereby improving the borrower's ability to pay. However, it results in an economic loss to the lender. To compensate for this loss, the lender takes a fractional interest in the home's equity, thereby gaining if and when home values recover. Therefore, the lender holds a share in a call option on the home value V, with a strike price of the loan amount L. Denoting this call option as $C(V, L)$, the lender will choose a fraction $\theta \in (0, 1)$ in the call such that it offsets the principal write-down, i.e., $\theta \cdot C(V, L) = W$. How does this affect the early-exercise value of the default option?

- We note that the loan balance has been reduced from L to $L - W$. This reduces the early-exercise value to the borrower and makes strategic default less likely.
- However, the borrower has given up a fraction θ of the home value, meaning that, instead of V in the early-exercise value, we now have $V - C(V, L) +$

Table 2

Logit regressions of re-default. The table presents logit regressions to explain the re-default rates of modified loans based on the modifications undertaken. A re-defaulted loan is one that is 90 days delinquent in the six-month period following the loan modification. The three variables that characterize the loan modification are the changes (old value minus the new value) in interest rate on the loan, the principal balance, and the maturity (term) of the loan. The other three variables in the logit regressions are instrumental variables. The loan-to-value ratio of the home (LTV) is an instrument that captures the moneyness of the default option, and captures the willingness to default. The debt ratio of the household is an instrument for the borrower's characteristics. The percentage payment change (PPC) is the percentage amount by which the monthly payment on the loan was reduced. This is an instrument for the improvement in the ability to pay of the borrower. The sample covers two years, 2007 and 2008, and results are reported separately by year. The columns in the table stratify the analysis into five monthly payment ranges, thereby looking at the data in terms of equal ability to pay after loan modification. The value to the right of the Wald statistic for the regression is the p-value. For parameter estimates, a chi-square statistic of greater than 3.84 is significant at the 5% level.

	Monthly Payment Amount after Modification									
2007	≤ 450		450-620		620-820		820-1120		≥ 1120	
	Est.	χ^2	Est.	χ^2	Est.	χ^2	Est.	χ^2	Est.	χ^2
Intercept	-2.91090	9.935	-3.23960	14.896	-1.75030	5.092	-2.71130	9.844	-1.47930	5.899
ΔRate	-0.11310	0.796	-0.28570	4.212	-0.31070	3.931	-0.33240	5.760	-0.36810	8.224
ΔTerm	-0.00640	5.427	-0.00133	0.220	0.00114	0.079	0.00016	0.002	-0.00038	0.008
ΔPrincipal	-0.00017	5.323	-0.00007	2.303	-0.00007	5.633	-0.00004	3.567	-0.00004	7.157
LTV	0.02270	6.143	0.02530	8.212	0.00163	0.040	0.02200	6.358	0.00141	0.043
Debt Ratio	-0.02110	2.021	-0.01060	0.802	-0.00291	0.056	-0.01790	3.182	-0.01150	1.357
PPC	-0.01000	0.421	-0.01340	1.888	-0.00324	0.033	-0.00463	0.106	0.00077	0.004
Re-Default	43		59		66		89		93	
No Default	230		330		382		459		545	
Wald Stat	18.9061	0.0043	18.8779	0.0044	15.1021	0.0195	23.4206	0.0007	20.8749	0.0019

	Monthly Payment Amount after Modification									
2008	≤ 450		450-620		620-820		820-1120		≥ 1120	
	Est.	χ^2	Est.	χ^2	Est.	χ^2	Est.	χ^2	Est.	χ^2
Intercept	-0.66310	3.595	-1.77870	18.532	-2.47180	39.303	-1.77290	22.438	-1.89300	39.145
ΔRate	0.13490	7.877	-0.16530	4.650	-0.01360	0.037	-0.30900	14.547	-0.14070	4.826
ΔTerm	0.00077	0.408	0.00028	0.042	0.00024	0.038	-0.00138	1.399	0.00035	0.143
ΔPrincipal	-0.00005	2.328	-0.00008	18.808	-0.00006	21.597	-0.00007	47.587	-0.00004	41.344
LTV	-0.00267	0.675	0.00806	4.207	0.01370	13.809	0.00653	3.627	0.00859	12.609
Debt Ratio	-0.01710	5.000	-0.00693	1.027	-0.00356	0.344	-0.00025	0.002	-0.00113	0.068
PPC	-0.02540	12.026	-0.00572	0.453	-0.01130	1.826	0.00540	0.417	-0.00754	1.286
Re-Default	177		205		286		375		522	
No Default	747		819		1000		1071		1564	
Wald Stat	32.3832	< 0.0001	42.1224	< 0.0001	46.1375	< 0.0001	83.5345	< 0.0001	81.2047	< 0.0001

$(1-\theta)C(V,L) = V - \theta C(V,L)$. That is, the borrower holds a complete position in the home until the value reaches L, after which it has sold a call on the home value V at strike L, and then bought back $(1-\theta)$ of a call.

- Assuming that the relocation costs K_R remain unaffected, the new early-exercise value is

$$(L - W) - [V - \theta C(V,L)] - K_R$$

Noting that $W = \theta \cdot C(V,L)$, we see that the early-exercise value is

$$[L - \theta \cdot C(V,L)] - [V - \theta C(V,L)] - K_R = (L - V - K_R)$$

which is the same as the early-exercise value without a SAM.

Hence, the incentive to exercise the put option to default remains unchanged when a SAM is implemented by means of a principal reduction. On the other hand, if a SAM is implemented with a rate reduction, then the early-exercise value is

$$L - [V - C(V,L) + (1-\theta)C(V,L)] - K_R = (L - V - K_R) + \theta C(V,L)$$

implying that the incentive to exercise the default put increases relative to a loan modification without a SAM.

Therefore, SAMs enhance the borrower's ability to pay, but may increase the willingness to re-default. Note too, that a SAM delays immediate foreclosure, and postpones imposition of the resultant deadweight costs. An ex-post recovery of the housing market, rather than a decline, makes a SAM efficacious.

Another desirable property of a SAM is that it discourages adverse selection against lenders. Strategic borrowers with negative equity in their homes may be unwilling to make loan payments even though they have the ability to do so. Such borrowers have every incentive to approach the lender for a principal reduction by pretending that they are distressed. However, if these borrowers are offered a SAM, they may be less willing to behave strategically, as they would have to part with some of their home equity in return for the principal write-down.

5 Comparison to current practice

The theoretical analysis so far has shown that, across schemes that are matched on ability to pay, principal write-down modifications maximize the willingness to pay, thereby maximizing the loan's economic value to the lender. This begs the question: What prescriptions are being followed in current practice? The OCC and OTS Mortgage Metrics Report (2009) covers 64% of all mortgages outstanding in the US, including subprime mortgages. They report various types of loan performance and loan modifications. The data covers 34 million loans with more than $6 trillion in principal balances, and covers their performance for five quarters ending in the first quarter of 2009. We abstract some of the results from this report for comparison to our model prescriptions.

The number of loan modifications has increased considerably from 68,001 in the 2008 Q1 to 185,156 in 2009 Q1, an almost three-fold increase. More than half the modifications entailed reduced monthly payments, i.e., A_{max} was reduced to accommodate distressed homeowners. Two-thirds of the modifications were combination modifications, entailing changes in two or more of loan rates, maturity, or adjustments to principal. These modifications are the opposite of what theory prescribes. A large fraction wrote *up* the loan balance by capitalizing missed payments (70.2%), 63.2% of modifications were effected by interest-rate reductions, and 25.1% extended maturity (p. 5 of the OCC report). Only 1.8% wrote down the principal balance as is theoretically optimal. As a consequence the "re-default rate" evidenced is extremely high. Of the loans modified in Q1 2008, 68% were foreclosed or delinquent a year later (p. 6).

Managing negative equity (or loan-to-value ratios) is very important in manipulating the borrower's willingness to pay. Foote, Gerardi, Goette and Willen (2009) find that a 10% fall in house prices raises the probability of delinquency by more than 50%. Liebowitz (2009) analyzes a database from McDash Analytics of 30 million loans and shows that it is not the features of the loan (subprime, Alt-A, etc.) that explain foreclosure, but the extent of negative equity in the home matters more. While only 12% of the loans had negative equity, they accounted for 47% of all foreclosures. More than half of foreclosed homes are funded by prime loans, and the foreclosure rate for prime-loan homes grew at twice the rate of subprime homes. Liebowitz also reports that the level of induced foreclosures from raising rates is small—rate increases did not lead to greater foreclosures unless they were greater than 4%. But most important, LTV should be lowered by writing down principal as prescribed by the preceding theory.

In short, by deviating from theoretical prescriptions, current loan modification practices may be dissipating value. What is the reason for this? Clearly, loan modifications have shied away from writing down principal, simply because the accounting impact would be much more negative than with other types of iso-service loan modifications. Adelino, Gerardi and Willen (2009) report that the incidence of principal reductions in the data is low, and this may be on account of adverse incentive effects, where borrowers who are not in danger of default may be induced to seek foreclosure in the hope of having their loan balances written down. Still, given that foreclosures have contagion effects on other homes in the neighborhood—see Harding, Rosenblatt and Yao (2008)—it is even more important that the tide of foreclosures be stemmed by optimal loan write-downs.

A simple examination of the right-hand-side plot in Figure 1 shows the wide range of loan values for the same level of monthly payment, after accounting for the incentive effects of strategic default by the borrower. Hence, the degradation from optimal loan value by choosing a suboptimal loan modification is quite severe. Roughly speaking Figure 2 shows that for a loan maturity of 30 years and a fixed annual servicing load of \$20,000 (A_{max}), if the loan rate is reduced from 6% to 5%, and the principal balance is written down less than is optimal at 6%, the loan value falls from about \$206K to about \$189K, a substantial decline in loan value of \$17K to the lender, even assuming no deadweight costs of foreclosure. If expected deadweight costs are taken into account, the decline in loan value to the lender is about \$20K if these costs are 20% of home value, and about \$25K if deadweight foreclosure costs are 40% of home value.

The consequence is that loan modifications that increase the likelihood of re-default relative to an optimal modification will incur expected deadweight costs of foreclosure as an additional penalty over and above that from the suboptimal modification per se. Hedge funds that invest in distressed home loans have recognized the optimality of principal write-downs. They buy loans from banks at deep discounts and are better able to manage the loan modification by writing down principal. Simon (2009) reports that—"Some mortgage investors have made principal reduction a part of their strategy, in part because it gives borrowers who owe more than their houses are worth an incentive to keep making payments. It is also easier to ultimately refinance or sell the mortgage if the borrower has equity." Principal reductions of \$100,000 or more are not uncommon.

6 Discussion and Concluding Comments

The analysis in this paper has shown that optimal modifications usually entail setting the loan rate on the mortgage equal to the current mortgage rate for the given borrower class, no extensions of maturity, and a writing down of the principal balance. These guidelines become more costly to violate when the lender's deadweight costs of foreclosure are high.

A suitable loan modification scheme must be cognizant of both, the borrower's ability to pay *and* willingness to pay. The former criterion is not that important, as shown empirically in Foote, Gerardi, Goette and Willen (2009). In contrast, the latter criterion, dependent on the incentive effects of the modification, accounts for large differences across loan modifications that are otherwise neutral to the borrower's ability to pay. Therefore, in this paper, we determine the optimal loan modification, by optimizing willingness to pay, after fixing the ability to pay.

The prescription to write down loan balances in a modification is bitter medicine for lenders, made especially so on account of accounting conventions that incentivize lenders to push losses into the future. But it is better than the palliatives being attempted in current practice. The shared appreciation mortgage (SAM) is an important innovation that deserves more detailed analysis in future work. We showed that SAMs improve the borrowers ability to pay, reduce the deadweight costs of foreclosure and mitigate adverse selection in loan modification programs.

We did not need to consider the impact of securitization on loan modification—our analysis begins when the decision to modify the loan has been taken, assuming it is permissible. However, servicer restrictions and incentives might vary for securitized loans, and the empirical evidence on whether securitization affects loan modification is quite mixed. Piskorski, Seru and Vig (2008) show that securitization induces a foreclosure bias on the part of servicers. Elul (2009) points out that securitized loans perform worse than non-securitized ones and therefore, we should observe more modifications of securitized loans. Adelino, Gerardi and Willen (2009) show that the empirical reluctance to renegotiate loans is not driven by securitization.

There is no doubt, however, that pooling and servicing agreements (PSAs) for securitized loans often prohibit changes to principal, which rules out the optimal channel for modification theoretically supported in this paper. Modification of securitized loans and the legal impediments to it are part of a larger game played between the homeowners, servicers and tranche holders in the securitization. Typically the AAA tranche, often comprising 80% of the collateral in the securitized pool will not agree to a loan modification that entails principal write-downs, even though it is optimal for them to do so as shown here. A possible approach would be to offer a government guarantee to these tranche holders if they would allow a modification, as from a societal point of view, this would stem the rate of foreclosures as well as eliminate the dead-weight costs of foreclosure.[5] Another approach to reducing foreclosure rates is to promise a principal write-down in a year or two, provided the homeowner makes monthly payments till then. The incentive effects of all these modifications are also an issue that we leave for further modeling in subsequent work.

We also did not need to consider prepayment risk commingled with default risk—as is done in Kau and Keenan (1999). A borrower in distress is not usually in a position to prepay the loan, hence the modification prescriptions are mostly unaffected by the consideration of prepayment risk.

[5] Suggested by Paul Kupiec of the FDIC.

While this paper has focused on mitigating adverse incentive effects on willingness to pay in loan restructuring, Foote, Gerardi, Goette and Willen (2009) point out that it is also important to institute reforms that will enhance the ability to pay, i.e., economic stimulus steps to raise employment and stem the fall in house prices. This long-term solution to the housing crisis must be complemented by short-term tactical loan modification based on the optimal principles developed in this paper.

References

Adelino, Manuel., Kristopher Gerardi, and Paul Willen (2009). "Renegotiating Home Mortgages: Evidence from the Subprime Crisis," Working paper, CEPR.

ambrose, brent., and Charles Capone (1996). "Cost-Benefit Analysis of Single-Family Foreclosure Alternatives," *Journal of Real Estate Finance and Economics* 13, 105-120.

Ambrose, Brent., Charles Capone, and Yongheng Deng (2001). "Optimal Put Exercise: An Empirical Examination of Conditions for Mortgage Foreclosure," *Journal of Real Estate Finance and Economics* 23(2), 213-234.

Anderson, Ronald., and Suresh Sundaresan (1996). "The Design and Valuation of Debt Contracts," *Review of Financial Studies* 9, 37-68.

Campbell, John., Stefano Giglio, and Parag Pathak (2008). "Forced Sales and House Prices," Working paper, Harvard University.

Cohen-Cole, Ethan., and Jonathan Morse (2009). "Your House or Your Credit Card, Which Would You Choose? Personal Delinquency Tradeoffs and Precautionary Liquidity Motives," Working paper, University of Maryland.

Cox, John C., Stephen A. Ross, and Mark Rubinstein. (1979). "Option Pricing: A Simplified Approach." *Journal of Financial Economics* 7, 229-263.

Deng, Yongheng., John Quigley, and Robert van Order (2000). "Mortgage Terminations, Heterogeneity and the Exercise of Mortgage Options," *Econometrica* 68(2), 275-307.

Elul, Ronel (2009). "Securitization and Mortgage Default: Reputation vs. Adverse Selection," Working paper, Federal Reserve Bank of Philadelphia.

Federal Deposit Insurance Corporation (2009). "FDIC Loan Modification Plan," Washington, D.C. `http://www.fdic.gov/consumers/loans/loanmod/loanmodguide.html`.

Foote, Christopher., Kristopher Gerardi, Lorenz Goette, and Paul Willen (2009). "Reducing Foreclosures: No Easy Answers," Working paper 2009-15, Federal Reserve Bank of Atlanta.

Guiso, Luigi., Paola Sapienza, and Luigi Zingales (2009). "Moral and Social Constraints to Strategic Default on Mortgages," Working paper, European University Institute.

Harding, John., Eric Rosenblatt, and Vincent Yao (2008). "The Contagion

Effect of Foreclosed Properties," Working paper, University of Connecticut.

Jarrow, Robert., and Andrew Rudd (1983). "Option Pricing," Irwin, Homewood Illinois.

Kau, James., and Donald Keenan (1999). "Patterns of Rational Default," *Regional Science and Urban Economics* 29, 217-244.

Kelly, Austin (2006). "Appraisals, Automated Valuation Models, and Mortgage Default," Working paper, Center for Economics, US Government Accountability Office, Washington, D.C.

Khandani, Amir., Andrew Lo, and Robert Merton (2009). "Systemic Risk and the Refinancing Ratchet Effect," Working paper 15362, NBER.

Liebowitz, Stan., (2009). "New Evidence on the Foreclosure Crisis," *Wall Street Journal*, July 3. `http://online.wsj.com/article/SB124657539489189043.html`.

Meadows, Ray (2009). "A Quantitative Comparison of Alternative Loan Modification Strategies," White paper, Recovery Partners, Inc.

Merton, Robert C. (1974). "On the Pricing of Corporate Debt: The Risk Structure of Interest Rates," *The Journal of Finance* 29, 449-470.

Mian, Atif., and Amir Sufi (2009). "House Prices, Home Equity-Based Borrowing, and the U.S. Household Leverage Crisis," Working paper, University of Chicago.

O'Brien, John., (2008). "Stabilizing the Housing Market," Working Paper, UC Berkeley.

Piskorski, Tomasz., Amit Seru, and Vikrant Vig (2008). "Securitization and Distressed Loan Renegotiation: Evidence from the Subprime Mortgage Crisis," Working paper, Columbia University.

Simon, Ruth., (2009). "Playing Mortgage Market Proves Tricky," *Wall Street Journal*, June 11, `http://online.wsj.com/article/SB124459303760100287.html`.

US Department of the Treasury (2009). "OCC and OTS Mortgage Metrics Report: Disclosure of National Bank and Federal Thrift Mortgage Loan Data (First quarter)," June, Washington, D.C.