Consumer Stockpiling and Competitive Promotional Strategies

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ABSTRACT

An examination of brand prices in several categories reveals that the distribution of prices is multi-modal with firms offering shallow and deep discounts. Another interesting feature of these distributions is that they may have holes in the interior of the support. These pricing distributions do not occur in extant theoretical models of price promotions. We develop a dynamic model of competition in which some price sensitive consumers stockpile during periods of deep discounts. A game-theoretic analysis of our model generates a multi-modal pricing distribution with a hole in the interior of the support. Consumer stockpiling in our model also gives rise to negative serial correlation in prices. This is consistent with our empirical observation of the pricing distribution of several brands across multiple categories in the IRI marketing dataset.

We generate several interesting insights into firms' optimal promotional strategies and their interplay with the clientele mix, market structure and other market factors. We find that in equilibrium stockpiling by price sensitive consumers neither harms nor benefit firms when they adopt equilibrium strategies. Interestingly, when price sensitive consumers stockpile, even increased consumption as a result of stockpiling does not lead to higher profits for firms.

Keywords: Promotional Strategies, Consumer Stockpiling, Endogenous Stockpiling Threshold, Interior Modes, Multiple Modes, Hole in Distribution, Negative Serial Correlation
1. Introduction

There is a rich literature in both economics and marketing which views price promotions as mixed strategy equilibrium (Shilony 1977, Varian 1980, Narasimhan 1988, Raju Srinivasan and Lal 1990, Rao 1991). The scant empirical work that examines whether price promotions are consistent with the mixed strategy interpretation has largely focused on testing whether price distributions of competing products exhibit independence (Rao, Arjunji and Murthi 1995). While Rao et. al. (1995) find that the prices across brands are independent, they note that in many cases distributions have more than two modes. Our examination of the distribution of prices of multiple SKUs across several categories in the IRI Marketing data set (Bronnenberg, Kruger and Mela, 2008) is consistent with their finding that the distribution of prices is often multi-modal and that prices frequently exhibit negative serial correlation.

![Price Distribution in a Frequently Purchased Category](image)

Figure 1: Price Distribution in a Frequently Purchased Category

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1 The distribution of prices is for a single SKU (Pepsi case of 24, 8 oz cans/bottles) between July 2001 and June 2002. The data is from a single store of a large chain in the Midwest. A time-series plot of the same SKU reveals a possible serial correlation in prices with no obvious trend or structural break. Please see Figure A1 in the appendix for a time series plot.
The salient features of a typical distribution of prices of a brand (a single stock keeping unit) in a frequently purchased category in a store are shown in the Figure 1. The x-axis in Figure 1 represents the range of prices and the y-axis represents the frequency (in percentage) which the various prices are charged. First, note that the pricing distribution is multi-modal. Second, often the modes are in the interior of the support and finally, there is a range of prices in the interior of the support which are not charged very often.\(^2\) The extant theory on price dispersion predicts a U shaped distribution which is bi-modal with the modes at the ends rather than in the interior of the support (as depicted in top left and right corners of Figure 1). One of the goals of this study is to offer an explanation for promotional policies that are consistent with the multi-modal distribution patterns and the negative serial correlation in prices documented in Tables A3 and A4 in the appendix.

Let us begin by examining the motivation and the consequence of offering price promotions. All else being equal, offering a price promotion lowers the margin. Consequently, in the absence of cost decreases and/or other externalities, firms can benefit only if demand expands in promotional periods. Heerde, Gupta, and Wittink (2003) and Steenburgh (2007) show that primary demand effects account for about two-thirds of the sales increase. It is therefore helpful to examine the sources of primary demand expansion. Primary demand expansion may occur if consumers increase their rate of consumption in promotional periods. However, in many categories such as toilet tissue, coffee etc. consumption rate is known to be relatively constant and so demand expansion in promotional periods is most likely a result of inter-temporal shifts in consumers’ purchases. Indeed this is consistent with the post-promotional dip in sales that is documented in numerous empirical studies.

We consider a symmetric oligopoly in which brands compete to serve the needs of consumers who differ in two dimensions: a) price sensitivity and b) propensity to stockpile. All consumers have the same reservation price and consume at a rate of one

\(^2\) Although this type of pricing distribution is pervasive we are unaware of any published work which documents this phenomenon. Please refer to the Table A3 in the appendix for summary of pricing distributions of several brands across several categories which exhibit multiple modes. The left hand side satellite picture in Figure 1 is the equilibrium pricing density function from Varian (1980) depicting utmost two modes. The right hand side satellite picture is the same from Raju, Srinivasan and Lal, (1990), depicting the presence of hole close to the reservation price.
unit each period. A fraction of the market is price insensitive and purchases a unit from one of the brands at random as long as the price is not greater than the reservation price. The remaining consumers are price sensitive and make purchases based solely on prices; they buy from the lowest priced firm. Consumers also differ in their ability to stockpile. Some consumers do not engage in stockpiling due to reasons such as high storage cost, space constraints or other non-strategic reasons. The remaining price sensitive consumers with low holding costs are willing to stockpile and purchase one additional unit for the future whenever prices are sufficiently low. Firms set prices to maximize discounted profits over an infinite horizon. We characterize the Markov Perfect Equilibrium of this game. The mixed-strategy equilibrium is state contingent; that is firms’ pricing strategies in any period depend not only on its pricing decisions but also on those of its rivals’ past decisions. More specifically, the pricing strategy in any given period depends critically on whether or not any brand has offered a deep enough promotion in the previous period to trigger stockpiling.

A novel feature of our model is that the price threshold below which stockpiling consumers engage in forward buying is determined endogenously. This is important because the promotional policies (depth and frequency of discounts) differ substantially across product categories. For instance, consider two categories one in which a discount of 50% off regular price is common while in the other category such discounts are seldom offered. Consumers may not stockpile in the first category when they see a 30% discount, but they may be willing to stockpile in the second category for the same discount. This implies that the firms’ pricing policies will have a bearing on the consumers’ stockpiling threshold, which in turn affects the quantity purchased. It is clear that firms’ optimal pricing policies should take into account the demand effects which depend on the stockpiling threshold. Therefore, in characterizing the equilibrium pricing policy the stockpiling threshold must be endogenously determined. Said differently, the firms must recognize that any change in market characteristics will not only affect their pricing policy directly but also indirectly through the effect on the stockpiling threshold.
Literature Review

Our work builds on two streams of literature: (a) the rich and established literature on price promotions/dispersion and (b) the recent but growing literature that analyzes consumers’ stockpiling behavior. Much of the extant literature on price promotion/dispersion (Shilony 1977, Varian 1980, Narasimhan 1988, Raju Srinivasan and Lal 1990, Rao 1991) proposes a static model to explain price dispersion, both contemporaneous and inter-temporal. Specifically, these studies show that there is no equilibrium in pure strategies. Given that stockpiling is intrinsically a dynamic problem we extend this literature by examining the pricing decisions of firms in an oligopoly where they maximize profits over an infinite horizon. The demand dynamics in our model are also distinct from the extant literature in this area. The aggregate demand in these models is assumed to be constant, so the short-term demand expansion resulting from promotions always comes at the expense of competing firms. However, recent empirical work by Heerde, Gupta, and Wittink (2003) and Steenburgh (2007) demonstrates that the aggregate demand expansion in promotional periods accounts for about two-thirds of the sales increase, which is consistent with the long established post-promotion dip in sales. In our model we allow some consumers to be forward looking and purchase for future consumption when they find the prices in the current period to be sufficiently attractive so that the aggregate demand in every period is not the same.

The second stream of research that our paper builds on is one that explicitly examines the impact of stockpiling on firms’ pricing decision. Before discussing the findings from this literature we would like to highlight certain institutional characteristics that we feel are important to incorporate in a model that examines consumers’ stockpiling behavior. First, the problem is inherently dynamic, as stockpiling by consumers in a given period will have an effect on their demand in the following period. This needs to be internalized by the firms in formulating their pricing strategies. Second, consumers seldom know future prices. While they may have some expectations of prices in future periods, they face uncertainty about the exact price offered in the future. Finally, given the dynamic nature of the decision problem (rational forward looking) consumers will stockpile only if the consumer surplus from stockpiling in a given period exceeds that
from not stockpiling or delaying purchase to the next period. This comparison of utility from stockpiling with the expected utility from not stockpiling yields a price threshold. If the current period price is lower than this threshold, consumers will prefer to stockpile. On the other hand, if the current period price is above this threshold there is no incentive to stockpile. These characteristics of the decision environment call for a model that is (a) dynamic, (b) yields equilibrium prices which are not deterministic so that consumers do not know the future prices with certainty and (c) explicitly incorporates the endogenous relationship between current and future period prices and its effect on triggering stockpiling behavior, thereby endogenously determining the stockpiling threshold.

Salop and Stiglitz (1982) consider a market in which consumers may stockpile and show that imperfect price information leads to an equilibrium in mixed strategies. Bell, Iyer and Padmanabhan (2002) consider a setting in which consumers may stockpile for future consumption but allow for flexible consumption so that consumers who stockpile may reenter the market with some probability. Both studies characterize the equilibrium mixing distribution which has two price points in its support. Anton and Das-Varma (2005) and Guo and Villas-Boas (2007) also consider settings in which consumers stockpile and characterize the pricing equilibrium which is in pure strategies. The paper that comes closest to our work is Hong, McAfee and Nayyar (2002) who consider an oligopoly setting in which firms compete for captives and shoppers. All shoppers in their model engage in stockpiling when prices fall below a certain threshold, which is exogenously determined. However, none of these theories can jointly explain pricing patterns that exhibit multiple modes, interior holes and negative serial correlation.

Our model takes into account a) consumers’ optimal response to promotions, b) firms’ optimal response to consumer stockpiling and c) competitive intensity in a single framework, which allows us to capture both the direct and the indirect effect of market characteristics on equilibrium pricing strategies. By doing so, our model not only generates predictions that are consistent with the pricing patterns observed in the frequently purchased categories but also delineates the conditions when stockpiling may (not) occur in equilibrium. The model also generates interesting insights on the optimal promotional strategies of firms and their interplay with the clientele mix, market structure and other market factors.
The remainder of the paper is structured as follows. We present the model in section 2. We begin with a description of the model primitives and develop some essential results in this section. We present our main results in section 3 where we highlight how the equilibrium outcomes vary with the clientele mix and the market structure. In section 4 we document some empirical regularities exhibited by the distribution of prices that are consistent with those documented in earlier work and demonstrate that our model predictions are consistent with the empirical observations. In section 5 we extend our base model to allow for increase in consumption after stockpiling and examine how it affects firms’ profits. We conclude with a discussion in section 6.

2. Model

We consider a market in which \( n \) symmetric firms, selling similar products, compete to maximize profits over an infinite horizon. All consumers have a reservation price of \( r \) for a unit of the product and consume one unit in each period but differ in their price sensitivity and holding cost.\(^3\) A fraction \( v \) of the consumers are price insensitive and purchase one unit each period from one of the \( n \) firms at random as long as the price is not greater than their reservation price \( r \). The remaining fraction of consumers of size \((1-v)\) are price sensitive and purchase the lowest priced product. Consumers are also heterogeneous in their ability to stockpile. A fraction \( \lambda \) of the consumers have a relatively low holding cost, assumed to be zero in this paper, and the rest have very high holding cost such that stockpiling is prohibitively expensive.

Since price insensitive consumers do not make purchase decisions based on prices, deep discounts do not affect their purchase decision. Price sensitive consumers with low holding cost, of size \((1-v)\lambda\), stockpile at deep discounts. We can think of these consumers as exhibiting forward-looking or strategic behavior.\(^4\) This is consistent with the empirical findings of Dillon and Gupta (1996) who find that the majority of the

\(^3\) In our model, without loss of generality all prices are normalized with respect to marginal cost, \( c \), and reservation price, \( r' \). The normalized price \( p = r.(p'-c)/(r'-c) \), where \( r' \), \( p' \) and \( c \) are the actual reservation price, the actual price and marginal cost respectively; \( r \) is any arbitrary constant. All this normalization does is, maps the \([r', c]\) space to \([r, 0]\) space.

\(^4\) We use the label forward-looking, strategic and stockpiling consumers interchangeably throughout this paper to refer to this segment.
increase in category sales during promotional periods comes from price sensitive consumers.

To keep the state-space manageable we assume that when forward-looking consumers stockpile they purchase only one additional unit.\(^5\) While there are an infinite number of periods there are only two possible states, \(s=\{0, 1\}\) in our model. The state \(s=0\) corresponds to the case in which none of the \(n\) firms offered a low enough price in the previous period to trigger stockpiling. Consequently, forward-looking consumers are left with no inventory in the current period. The state \(s=1\) corresponds to the case when at least one of the \(n\) firms offered a deep enough discount to trigger stockpiling in the previous period, such that strategic consumers purchased an additional unit for future consumption leaving them with an inventory of one in the current period. The example in the figure below helps to clarify the state-space and demand dynamics.

![State-Space Diagram](image)

Figure 2: Forward-Looking Consumers’ Behavior

Consider the market at time \(t\) and suppose forward-looking consumers have no inventory so that \(s=0\). If none of the \(n\) firms offer a price low enough to trigger stockpiling (high price branch in the figure) they purchase one unit for consumption in period \(t\) and end up with no inventory \((s=0)\) in \(t+1\). In period \(t+1\), they are faced with the same decision problem. Now suppose at least one of the \(n\) firms offers a deep enough discount in \(t+1\) so that these consumers stockpile. In our model when consumers stockpile in \(s=0\), they purchase 2 units: one for current consumption and one for future consumption.

\(^5\) Niraj, Padmanabhan and Seetharaman (2008) find that 95% of quantity purchases are concentrated on one or two units.
consumption. Now consider the case when these consumers have an inventory of one unit at time \( t \) (state \( s=1 \)), if no firm offers a deep enough discount then these consumers simply consume the product in their inventory and do not purchase. On the other hand, if one of the firms offers a deep enough discount they stockpile but given that they have an inventory of one unit for current consumption, they simply purchase one unit for future consumption. Thus, the key difference in \( s=0 \) and \( s=1 \) is that at high prices strategic consumers purchase 1 or 0 unit respectively, but at deep discounts they purchase 2 or 1 unit respectively.

A relevant question to ask is whether the discount that is required to trigger stockpiling (low price in Figure 2) depends on the state of consumers' inventory? We formally analyze the consumers' decision of whether or not to stockpile at any given price in each state and show (in Lemma 1) that in our model the stockpiling threshold does not depend on the state. Let \( U_s \) denote the expected continuation utilities of strategic consumers net of price at the time of purchase in state \( s, s=0, 1 \) and \( \delta_c \) denote the consumers’ discount factor.

**Lemma 1**: There exists a single price threshold, \( t \), regardless of strategic consumers’ state of inventory, such that it is optimal for strategic consumers to stockpile for the next period whenever the minimum price in the market is less than or equal to the threshold:

\[
p^*_s \leq t = \delta_c r - \delta_c (U_0 - U_1)
\]

**Proof**: Please see Appendix.

The strategic consumer’s decision problem is whether to buy an additional unit for future consumption at current minimum price and move to (or remain in) state \( s=1 \) in the next period or not to buy for future consumption and move to (or remain in) state \( s=0 \) in the next period. It is helpful to rewrite the above condition as \( \delta_c r - p^*_s \geq \delta_c (U_0 - U_1) \).\(^6\) The left hand side of this inequality, \( \delta_c r - p^*_s \), denotes the benefit (discounted consumer

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\(^6\) \( U_0 \) and \( U_1 \) are explicitly characterized in terms of firms’ pricing strategies in (11) and (12). The formal condition that the equilibrium stockpiling threshold must satisfy is defined in (13). Since \( U_s \) represents the continuation utilities at the time of purchase \( U_0 \) is always greater than \( U_1 \) because \( U_0 \) accounts for the possibility of buying and consuming an extra unit.
surplus) from buying for future at the current minimum price and the right hand side
denotes the cost of stockpiling, moving from state \( s=0 \) to state \( s=1 \). Note to compute the
stockpiling threshold we need to know the continuation utilities \( U_0 \) and \( U_i \) which in turn
depend on the firms' pricing strategies. Consequently, we now analyze the firms' decision
problem.

Given that firms are symmetric, let \( V_s \) denote the expected continuation payoff
to firms in state \( s, s=\{0, 1\} \) and let firms' discount factor be \( \delta_F \).

**Demand Characterization**

After establishing the unique stockpiling threshold for both states in lemma 1, we
can now compute the demand facing firms in each state for a given stockpiling threshold,
\( t \). Let \( D_s(p, p_{-i}) \) denote the demand facing firm \( i \) in state \( s \), given the vector of
prices \( p_{-i} \), charged by all the other firms. Since the behavior of strategic consumers in
any given state depends upon how the price compares to the stockpiling threshold \( t \) we
characterize the demand for each state conditional on these events.

\[
D_0(p_i, p_{-i} | p_i > t) = \begin{cases} 
\frac{\nu}{n} + \frac{1-\nu}{n} & \text{if } p_i < \min(p_{-i}) \\
\frac{\nu}{n} & \text{if } p_i > \min(p_{-i}) 
\end{cases}
\]

(1)

If the focal firm charges a price greater than \( t \), in state \( s=0 \) strategic consumers will
purchase a maximum of one unit. If the focal firm’s price is the lowest then it sells to a
fraction \( \nu/n \) of the price insensitive consumers and all \( (1-\nu) \) price sensitive consumers. If
it is not the lowest priced firm then it only sells to price insensitive customers. Next we
specify the demand in \( s=0 \) when the focal firm charges a price below the stockpiling
threshold;

\[
D_0(p_i, p_{-i} | p_i \leq t) = \begin{cases} 
\frac{\nu}{n} \left(1-\nu(1+\lambda) \right) & \text{if } p_i < \min(p_{-i}) \\
\frac{\nu}{n} & \text{if } p_i > \min(p_{-i}) 
\end{cases}
\]

(2)

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7 Notation is summarized in the Table A1 in the appendix.
If the focal firm has the lowest price, then the forward-looking consumers of size \((1 - \nu) \lambda\) stockpile and purchase two units, while the remaining price sensitive consumers of size \((1 - \nu)(1 - \lambda)\) purchase one unit resulting in a demand of 
\[
2 (1 - \nu) \lambda + (1 - \nu)(1 - \lambda) = (1 - \nu)(1 + \lambda) \text{ units.}
\]
In addition the firm sells to a fraction \(\nu/n\) of the price insensitive consumers. If it is not the lowest priced firm then it sells only to the price insensitive consumers. The demand in \(s=1\) is computed in a similar fashion:

\[
D_i(p_i, p_{-i} | p_i > t) = \begin{cases} 
\frac{\nu}{n} + (1 - \nu)(1 - \lambda) & \text{if } p_i < \min(p_{-i}) \\
\frac{\nu}{n} & \text{if } p_i > \min(p_{-i}) 
\end{cases}
\]

\[
D_i(p_i, p_{-i} | p_i \leq t) = \begin{cases} 
\frac{\nu}{n} + (1 - \nu) & \text{if } p_i < \min(p_{-i}) \\
\frac{\nu}{n} & \text{if } p_i > \min(p_{-i}) 
\end{cases}
\]

Given the discontinuities in demand in (1) to (4) as in Varian (1980) and Narasimhan (1988) there is no equilibrium in pure strategies. Consequently, we proceed with the characterization of the symmetric Markov Perfect Equilibrium of this game.\(^8\)

**Equilibrium Analysis**

Let \(F_i(p)\) and \([l_o, r]\) respectively denote the equilibrium mixing distribution and corresponding support of any firm in state \(s\). Given (1) and (2) the equilibrium mixing distributions in \(s=0\) must satisfy the following conditions:

\[
p \left[ \frac{\nu}{n} + (1 - \nu)(1 - F_0(p))^{(n-1)} \right] + \delta_r V_0 \left(1 - F_0(t)\right)^{(n-1)} + V_i \left(1 - (1 - F_0(t))^{(n-1)} \right) = V_0 \quad \forall p \in (t, r]
\]

\[
p \left[ \frac{\nu}{n} + (1 - \nu)(1 + \lambda)(1 - F_0(p))^{(n-1)} \right] + \delta_r V_i = V_0 \quad \forall p \in [l_0, t]
\]

Equations (5) and (6) require that any price charged in the support must yield the same expected profit in equilibrium. Specifically, in \(s=0\) the firms’ expected profit for any

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\(^8\) Symmetric equilibrium is also the unique sub-game perfect equilibrium among all possible symmetric and asymmetric equilibria (Baye et. al. 1992).
price in the support of $F_0(p)$ is $V_0$. Equations (5) and (6) also recognize the difference in behavior of strategic consumers when firms charge prices higher or lower than the stockpiling threshold, $t$. Following (1) the first term in the left hand side of (5) represents the expected profit from charging a price greater than $t$. The firm is guaranteed price insensitive consumers of size $\frac{\nu}{n}$ but price sensitive consumers of size $(1-\nu)$ purchase its product only if it happens to be the lowest priced firm which happens with probability $(1-F_0(p))^{(n-1)}$. The second term represents the continuation payoffs. Given that the focal firm is charging a price higher than $t$ the state in the next period will depend upon rival firms’ pricing strategies in the current period. If none of the rival firms charges a price lower than $t$, an event which occurs with probability $(1-F_0(t))^{(n-1)}$, strategic consumers do not have the opportunity to stockpile and transition to $s=0$ where the firms’ continuation payoff is $V_0$. With complementary probability, $1-(1-F_0(t))^{(n-1)}$ at least one of the firms charges a price lower than $t$ allowing consumers to stockpile and transition to $s=1$ with firms’ continuation payoff of $V_1$. The second term in (5) thus represents the firm’s expected continuation payoff given the probabilities of transitioning to $s=0$ or 1.

The left hand side of (6) defines the expected profit of firms in $s=0$ from charging any price less than or equal to $t$ in the support. The first term follows from (2) and the second term represents the continuation payoffs. In contrast to (5) given that the focal firm charges a price lower than $t$, transition to $s=1$ is guaranteed and is independent of rival firms’ pricing decisions. Equations (7) and (8) similarly require that the expected payoffs from charging any price in the support in $s=1$ are $V_1$ and follow from (3) and (4) respectively.

\[
p \left[ \frac{\nu}{n} + (1-\nu)(1-\lambda)(1-F_1(p))^{(n-1)} \right] \\
+ \delta_r \left[ V_0 (1-F_1(t))^{(n-1)} + V_1 \left( 1-(1-F_1(t))^{(n-1)} \right) \right] = V_1 \quad \forall p \in (t, r) \tag{7}
\]

\[
p \left[ \frac{\nu}{n} + (1-\nu)(1-F_1(p))^{(n-1)} \right] + \delta_r V_1 = V_1 \quad \forall p \in [l, t] \tag{8}
\]
**Proposition 1:** Firms’ continuation profits are independent of state and the proportion of stockpiling price sensitive consumers: 

$$V_0 = V_1 = \frac{rv}{n(1-\delta_f)}$$

**Proof:**

Substitute $p = r$ in (5) and (7) to get

$$\frac{V_r}{n} + \delta_F \left[ V_0 \left(1 - F_0(t)\right)^{(n-1)} + V_1 \left(1 - (1 - F_0(t))^{(n-1)}\right) \right] = V_0$$

$$\frac{V_r}{n} + \delta_F \left[ V_0 \left(1 - F_1(t)\right)^{(n-1)} + V_1 \left(1 - (1 - F_1(t))^{(n-1)}\right) \right] = V_1$$

We solve the above equations simultaneously to obtain $V_0$ and $V_1$.

**QED**

We find that equilibrium profits are not only independent of state but also independent of the proportion of stockpiling consumers, $\lambda$. Interestingly, the equilibrium profits (given the equilibrium mixing distributions characterized here) are identical to the case in which price sensitive consumers do not stockpile. In other words, if we set $\lambda = 0$, then our model is identical to the dynamic version of Varian (1980) where the equilibrium profits are identical to that stated in Proposition 1. What this means is that if price sensitive consumers stockpile and firms’ adopt equilibrium strategies then firms’ can obtain the same profits as they would obtain in a world where there was no stockpiling.

Before characterizing the equilibrium mixing distribution we provide the rationale for the existence of dominated prices (hole) in the interior of the support. If the stockpiling threshold is in the interior of the support then as noted earlier, firms will face different demand at prices above and below the stockpiling threshold, $t$, in each state. This implies that firms' mixing distribution will be different for prices above and below the stockpiling threshold, $t$. To see this substitute $V = V_0 = V_1$ from Proposition 1 in (5) and (6) to obtain:

$$p \left[ \frac{V_r}{n} + (1-\nu)(1 - F_0(p))^{(n-1)} \right] + \delta_f V = V \quad \forall p \in (t, r]$$

$$p \left[ \frac{V_r}{n} + (1-\nu)(1 + \lambda)(1 - F_0(p))^{(n-1)} \right] + \delta_f V = V \quad \forall p \in [l_0, t]$$

(9)
We denote the cumulative distribution function above and below $t$ respectively by superscript 'a' and 'b' in each state $s=\{0,1\}$

$$F^a_s(p) \quad \forall p \in (t,r]$$

$$F^b_s(p) \quad \forall p \in [l_s,t]$$

To verify the existence of dominated prices notice that the cumulative distribution function should be non-decreasing which implies that at the stockpiling threshold, $t$, $F^b_s(t) \leq F^a_s(t)$ must hold. However, using the equi-profit condition in (9) it is easy to see that $F^a_s(t) < F^b_s(t), \forall \bar{\lambda} > 0$, which violates the non-decreasing property of the cumulative distribution function. This contradiction can only be resolved if prices in the neighborhood of $t$ are not in the support of the equilibrium mixing distribution. We establish this formally in Lemma 2.

**Lemma 2** There is a hole in the firms' mixing distribution whenever the stockpiling threshold is in the interior of the support of the mixing distribution in any state. In state $s=0$ the hole extends from stockpiling threshold, $t$ to $h_0 = t \left( \frac{r + r\bar{\lambda}}{r + t\bar{\lambda}} \right)$ and in state $s=1$ the hole extends from stockpiling threshold, $t$ to $h_1 = t \left( \frac{r}{r - \bar{\lambda}(r-t)} \right)$.

**Proof:** See appendix

To understand the intuition behind this result, note that prices at or below $t$ result in a discontinuous increase in demand in both states.\(^9\) Consider the case when a firm charges $t$ while all other firms charge a price strictly greater than $t$. In equilibrium the firm’s expected profit from charging $t$ and $t+\varepsilon$ must be equal. Given the discontinuous increase in demand due to consumer stockpiling at prices at or below $t$, the price $t+\varepsilon$ is strictly dominated. Figure 3 illustrates the cumulative distribution function of prices in any state $s=\{0, 1\}$ when the stockpiling threshold, $t$, is in the interior of the support.

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\(^9\) Notice that as $\bar{\lambda} \to 0$, $h_0 \to t$ and $h_1 \to t$. In other words in the absence of stockpiling price sensitive consumers the hole disappears.
Figure 3: Cumulative Distribution Function in state $s = \{0, 1\}$ with stockpiling

The cumulative distribution of prices in any state $s$ has two intervals of prices: $[l_s, t]$ and $[h_s, r]$ which firms charge with positive probability. The former interval denotes deep discounts that trigger stockpiling behavior. Prices in the interval $[h_s, r]$ represent shallow discounts that firms offer to compete for price sensitive consumers without triggering any stockpiling. The third interval, $(t, h_s)$, is the flat region of the CDF (or the hole) with zero mass and represents the interval of prices that firms will not charge in equilibrium. The mixing distribution takes this form when the stockpiling threshold, $t_s$, is in the interior of the support.

We present the mixing distribution $F_s(p | t)$ for a given stockpiling threshold, $t$ in each state in (10) when $t$ is in the interior of the support. The details can be found in the appendix.

$$
F_s(p | t) = \begin{cases} 
1 & , r \leq p \\
1 - \left[ \frac{v(r-p)}{np(1-v)} \right]^{1-\frac{1}{r+t\lambda}} & , \frac{rt(1+\lambda)}{r+t\lambda} \leq p < r \\
1 - \left[ \frac{v(r-t)}{np(1-v)(1+\lambda)} \right]^{1-\frac{1}{r+t\lambda}} & , t \leq p < \frac{rt(1+\lambda)}{r+t\lambda} \\
1 - \left[ \frac{v(r-p)}{np(1-v)(1+\lambda)} \right]^{1-\frac{1}{r+t\lambda}} & , \frac{rv}{v+n(\lambda+1)(1-v)} \leq p < t \\
0 & , p < \frac{rv}{v+n(1-v)} \\
\end{cases} 
$$

$$
F_t(p | t) = \begin{cases} 
1 & , r \leq p \\
1 - \left[ \frac{v(r-t)}{np(1-v)} \right]^{1-\frac{1}{r(t(1-\lambda)+t\lambda)}} & , \frac{rt}{r(1-\lambda)+t\lambda} \leq p < r \\
1 - \left[ \frac{v(r-t)}{np(1-v)} \right]^{1-\frac{1}{r(t(1-\lambda)+t\lambda)}} & , \frac{rv}{v+n(1-v)} \leq p < t \\
1 - \left[ \frac{v(r-t)}{np(1-v)} \right]^{1-\frac{1}{r(t(1-\lambda)+t\lambda)}} & , p < \frac{rv}{v+n(1-v)} \\
\end{cases} 
$$

(10)
The interested reader can easily verify from (10) that as \( \nu \to 1 \) the lower bound in both states go to reservation price, \( l_s \to r \), and the equilibrium mixing distribution degenerates to a single price \( r \). In this case firms do not offer promotions and always charge the reservation price, \( r \), as there are no price sensitive consumers to compete for in the market. On the other hand, as \( n \to \infty \) or \( \nu \to 0 \), \( F_0(p \mid t) \to 1 \) and \( F_1(p \mid t) \to 1 \) for all positive prices \( p > 0 \). In other words when all consumers are price sensitive, the pricing distribution degenerates to marginal cost which is normalized to zero in our model. This is the well known Bertrand result where firms earn zero profits in equilibrium.\(^{10}\)

Given (10), we can now analyze the strategic consumers' decision problem and compute the continuation utilities which will allow us to endogenously determine the stockpiling threshold.

**Endogenous Stockpiling Threshold**

Recall from lemma 1 that the stockpiling threshold \( t = \delta r + \delta_e U_1 - \delta_e U_0 \) is a function of the strategic consumers’ expected continuation utilities in both states 0 and 1. Specifically, the expected continuation utility of strategic consumer at the time of purchase in state \( s=0 \) is:

\[
U_0 = (1 - F_0(t))^n \left( r - E_{t < p_0^\text{min}} \left[ p_0^\text{min} \right] + \delta_e U_0 \right) + (1 - F_0(t))^n \left( 1 + \delta_e \right) r - 2 E_{p_0^\text{min} \leq t} \left[ p_0^\text{min} \right] + \delta_e U_1 \]

\( (11) \)

\(^{10}\) We thank an anonymous AE for encouraging us to generate the known results by varying \( \nu \) to bolster the face validity of our findings.
and that in state $s=1$ is:

$$U_1 = (1 - F_1(t))^n \left( \delta_c U_0 \right) + \left( 1 - (1 - F_1(t))^n \right) \left( \delta_c r - E_{p \in s=1} \left[ p_{i \min}^{\min} \right] + \delta_c U_1 \right)$$

(12)

here expected minimum prices in state $s = \{0, 1\}$ are:

$$E_{p \in s=1} \left[ p_{i \min}^{\min} \right] = \int_{t} npf_s(p) \left( 1 - F_s(p) \right)^{n-1} dp,$$

and $E_{t < p \in s=1} \left[ p_{i \min}^{\min} \right] = \int_{h_{t}(t)} npf_s(p) \left( 1 - F_s(p) \right)^{n-1} dp$

The expected continuation utility of strategic consumers in each state depends on whether or not they have had the opportunity to stockpile.\(^{11}\) The first term on the RHS of (11) and (12) represents the expected utility in the event that all firms charge a price above the stockpiling threshold. In (11), when the lowest price is greater than $t$, strategic consumers purchase one unit, pay the expected minimum price, and transition to $s=0$ in the next period with continuation utility of $\delta_c U_0$. In (12), when the lowest price is greater than $t$, strategic consumers do not purchase as they already have inventory for current consumption and transition to $s=0$ in the next period with continuation utility of $\delta_c U_0$. The second term on the RHS of (11) and (12) represents the expected utility in the event that at least one firm charges a price less than the stockpiling threshold. In $s=0$, strategic consumers purchase two units, pay the expected minimum price conditional on the fact that the price is less than $t$ and transition to $s=1$ in the next period with a continuation utility of $\delta_c U_1$. In $s=1$, consumers purchase one unit as they have inventory for current consumption and transition to $s=1$ in the next period with continuation utility of $\delta_c U_1$. Using (11) and (12) in conjunction with Lemma 1 we state the equilibrium condition for the stockpiling threshold, $t^*$, in terms of firms’ mixing distribution characterized in (10). The stockpiling threshold is the solution to the following equation.

$$t^* = \frac{\delta_c \left( (1 - F_0(t))^n E_{t < p_0 \in s=1} \left[ p_{0 \min}^{\min} \right] + 2(1 - (1 - F_0(t))^n) E_{p_0 \in s=1} \left[ p_{0 \min}^{\min} \right] - (1 - (1 - F_1(t))^n) E_{p_{i \min} \in s=1} \left[ p_{i \min}^{\min} \right] \right) \left( 1 - (1 - F_0(t))^n \right)}{1 + \delta_c \left( (1 - F_1(t))^n - (1 - F_0(t))^n \right)}$$

(13)

This completes the characterization of the equilibrium distributions.

\(^{11}\) Effects of exogenous market parameters on continuation utilities are reported in Table A3 in the appendix.
Endogenizing the stockpiling threshold by explicitly accounting for strategic behavior on the part of consumers presents non-trivial methodological challenges. It is reasonable therefore to ask what the value is of engaging in this exercise. Are the predictions from our model any different from a model that does not endogenize the threshold? We have shown that in a market where consumers are willing to stockpile in equilibrium the firm would offer shallow and deep discounts. As a practical matter a manager may be interested in the frequency and depth of discounts and how these metrics vary with market parameters. If we let $M$ denote a metric of interest and ask how $M$ is affected by a market parameter $\theta$, then to obtain the net effect we need to examine:\(^{12}\)

$$\frac{dM}{d\theta} = \frac{\partial M}{\partial \theta} + \frac{\partial M}{\partial t} \frac{\partial t}{\partial \theta}.$$  

The first term on the right hand side is the direct effect of the market parameter on the metric of interest and the second term represents the indirect effect of the market parameter on $M$ through its effect on the stockpiling threshold. If the stockpiling threshold is not endogenous, then the indirect effect $\frac{\partial M}{\partial t} \frac{\partial t}{\partial \theta} = 0$. Consequently, depending on the sign of $\frac{\partial M}{\partial t} \frac{\partial t}{\partial \theta}$ we may over/under estimate the effect of the market parameter on the metric of interest. Given that the equilibrium is fully characterized we now present the main results in the next section.

3. Results

In (10) we characterize the mixing distributions in each state conditional on a given stockpiling threshold, $t$. The stockpiling threshold in equilibrium is endogenously determined and is the solution to (13) which is a transcendental equation. All the parameters of interest in our model, with the exception of the number of firms, $n$, lie in a compact set. Specifically, $v, \lambda$ and $\delta$ all lie in the $[0,1]$ interval and as noted earlier the reservation price, $r$, can be normalized to any arbitrary constant without loss of generality (please refer to footnote 3). By choosing different values of the market parameters over the compact set we are able to characterize the equilibrium strategies under all market

\(^{12}\) The metric of interest may include frequency and average depth of deep or shallow discounts, the average price in each state etc.
conditions. We begin by highlighting the role of the clientele mix and market structure on the equilibrium outcomes.

Note from (10) that the lower bound in state $s=0$ is lower than that in state $s=1$, $l_0<l_1$. Depending on market conditions it is possible that the endogenous equilibrium threshold, $t^*: l_0<t^*<l_1$, is below the lower bound in $s=0$ in which case there will be no stockpiling in either state in equilibrium. It is also possible that $t^*: l_0<t^*<l_1$ such that it is below the lower bound of the support in $s=1$ but greater than the lower bound in $s=0$. In this case, stockpiling can occur only when consumers do not have an inventory. Figure 4 highlights market conditions under which stockpiling may (not) result in equilibrium.

In Figure 4, the $y$-axis represents the proportion of price insensitive consumers, $\nu$, while the $x$-axis represents the size of the segment willing to stockpile, $\lambda$. Panel $a$ represents a duopoly market, while panel $b$ represents an oligopoly with 5 firms, $n=5$. The effect of market structure on the equilibrium outcome is best understood by comparing the two panels in Figure 4. As expected, the parameter space over which stockpiling occurs is larger when competition is more intense in panel $b$ vis-à-vis panel $a$. In both cases, consistent with intuition, we find that when the size of the price insensitive consumer segment, $\nu$, is large, stockpiling does not occur in equilibrium. In contrast, when $\nu$ is not too large then stockpiling is more likely to result in equilibrium.

Interestingly, we find that when $\nu$ is not too large as the proportion of stockpiling consumers, $\lambda$, increases, stockpiling is more likely to occur only in the state in which

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Discount factors are fixed at $\delta_c=\delta_F=0.8$
consumers have no inventory, \( s=0 \) (see Panel a above). The rationale for this is that when the proportion of consumers willing to stockpile is large, firms compete more intensely in periods when they do not have inventory. In state \( s=0 \) the expected benefit resulting from increased demand offsets the cost of acquiring their sales at deep discounts. In the subsequent period after stockpiling, when the clientele mix gets skewed towards more price insensitive consumers, firms harvest by charging higher prices. Indeed for values of \( \nu \) exceeding a certain threshold it is sub-optimal for firms to charge prices less than \( t^* \) in \( s=1 \), as the cost of subsidizing the price insensitive consumers and the price sensitive consumers who purchase only for current consumption dominates the expected benefit of the demand expansion that may result from strategic consumers who purchase for future consumption. Thus, when \( \nu \) is not too large but \( \lambda \) is large firms compete intensely to sell to strategic consumers when they do not have inventory and harvest in the subsequent period.\(^\text{14}\)

**Equilibrium Pricing Strategy**

So far we have delineated the effect of the clientele mix and market structure on the equilibrium outcome. We now present how the conditional and unconditional expected prices, the depth and frequency of discounts and the equilibrium stockpiling threshold vary with respect to market parameters. In particular, we will highlight the role of consumers’ discount factor (\( \delta_c \)) and the proportion of price sensitive strategic consumers (\( \lambda \)) on the equilibrium promotional strategies.\(^\text{15}\) For practitioners this offers some guidance on how the depth and frequency of shallow and deep promotions should be adjusted to different market conditions. In the left panels in figures below, \( E1, E0, E1_{\text{min}} \) and \( E0_{\text{min}} \) (\( E \) and \( E_{\text{min}} \)) denote the conditional (unconditional) expected prices and the expected minimum prices. In the right panels, \( Pr(s=0) \) denotes the unconditional probability that strategic consumers have low inventory, \( t \) denotes the stockpiling threshold, \( F0(t) \) and \( F1(t) \) denote probabilities of firms charging prices below

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\(^{14}\) We thank an anonymous reviewer for encouraging us to highlight this interesting result.

\(^{15}\) Firms’ discount factor does not affect equilibrium outcomes except the value function \( V_c \).
the stockpiling threshold in state $s=0$ and $s=1$ respectively and $F(t)$ denotes the unconditional probability of firms charging prices below the stockpiling threshold.\footnote{Please refer to state transition probability section in the appendix for details on how unconditional probabilities are obtained.}

We begin by highlighting the effect of consumers’ discount factor on the firms’ equilibrium strategies. As one might expect stockpiling does not occur in equilibrium when consumers’ discount factor is low. As consumers’ discount factor increases first they only stockpile in the low inventory state, $s=0$, and when the discount factor is sufficiently large stockpiling occurs with positive probability in both states.

![Equilibrium outcomes with respect to consumer discount factor, $\delta_c$.](image)

Figure 5: Equilibrium outcomes with respect to consumer discount factor, $\delta_c$.\footnote{Figure 5 is a representative picture and is drawn based on exogenous parameter values of $r=1$, $n=3$, $\lambda=0.5$, $\psi=0.5$ and $\delta_f=0.9$. The x axis in Figure 5 represents the consumer discount factor, $\delta_c$, and the y axis represents the relevant metric of interest.}

Not surprisingly, the stockpiling threshold, $t$, increases with consumers’ discount factor. What is surprising is that in equilibrium with stockpiling the unconditional expected ($E$) and expected minimum prices ($E_{\text{min}}$) are increasing even though the expected prices in each state are decreasing in the consumers’ discount factor (see left panel in Figure 5). The rationale behind this finding is that the stockpiling threshold and the probability of firms charging prices below the stockpiling threshold in each state, $F0(t)$ and $F1(t)$, are all increasing in consumers’ discount factor (see right panel Figure 5). This means that when consumers’ discount factor is high, firms are more likely to offer deep discounts and the probability that consumers will end up in the high inventory state, $s=1$ is greater relative to that when discount factor is low. Consequently, although the expected prices in each state ($E0$ and $E1$) are decreasing, the unconditional expected price ($E$) will increase, because the expected price in $s=1$ is higher than that in $s=0$ and consumers are more often in state $s=1$ than in $s=0$ as $\delta_c$ increases. We would to like to
highlight that none of these findings could have been obtained without endogenizing the stockpiling threshold.  

We now turn to the effect of $\lambda$, the proportion of stockpiling price sensitive consumers, on the firms' equilibrium strategies. One might expect that as the proportion of stockpiling consumers increase in the market firms would compete more intensely for their business resulting in lower expected prices, $(E)$. Interestingly counter to this intuition firms’ expected prices increase with the proportion of stockpiling consumers.

![Figure 6: Equilibrium outcomes with respect to the proportion of stockpiling consumers, $\lambda$.](image)

This occurs because in the state with low inventory, $s=0$, as $\lambda$ increases the potential demand from stockpiling increases and firms compete more intensely for this demand, consequently expected prices, $(E0)$ and $(E0\_min)$ decrease in $\lambda$ (see left panel in Figure 6). Furthermore, if consumers’ stockpile in $s=0$ they transition to $s=1$ where the expected minimum prices are higher $(E1\_min > E0\_min$ and $E1 > E0)$ and increasing in $\lambda$. In markets with higher $\lambda$ the firm’s ability to charge higher prices in periods after stockpiling increases significantly and more than offsets the decrease in price resulting from intense price competition in low inventory states. Hence the unconditional expected prices $(E)$ are higher in markets with larger proportion of stockpiling consumers.

We would also like to note that as the proportion of strategic price sensitive consumers, $\lambda$, increases, the stockpiling threshold decreases. The intuition for this is that

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18 Note that if the stockpiling threshold were exogenous, the unconditional expected prices would be obtained by pricing distributions characterized in equation (10), which are independent of the consumers’ discount factor.

19 Figure 6 is a representative picture and is drawn based on exogenous parameter values of $r=1$, $n=2$, $\nu=0.25$, $\delta_c=0.9$ and $\delta_f=0.9$. The x axis in Figure 6 represents the proportion of stockpiling consumers, $\lambda$, and the y axis represents the relevant metric of interest.
stockpiling consumers can remain in the state of low prices, \( s=0 \), if they do not stockpile. Said differently, consumers must be offered a sufficiently good deal to exit out of this state and enter the high inventory state where the expected minimum prices \( (E_{1\text{ min}}) \) are significantly higher. As the proportion of strategic price sensitive consumers, \( \lambda \), increases the difference in \( E_{1\text{ min}} \) and \( E_{0\text{ min}} \) increases, and consequently the stockpiling threshold decreases.

The above discussion highlights the sensitivity of the equilibrium outcome and the optimal promotional strategies to various market conditions and can serve as a guide to practitioners. In the next section we highlight pricing patterns predicted by the model and demonstrate that they are consistent with commonly observed pricing patterns.

4. Empirical Regularities and Model Predictions

Recall from Figure 1 that the pricing distribution often is multi-modal with modes and hole in the interior of the support. With the exception of Rao, Arjunji and Murthi (1995), we are unaware of any other paper that documents the prevalence of multiple modes. Hence, in section 4.1, we first document our empirical findings as they pertain to the prevalence of multiple modes in pricing distributions using data on multiple brands across several product categories. We also demonstrate that our model predictions are consistent with these commonly observed patterns. Another empirical regularity is that the serial correlation in prices is often negative. In 4.2 we empirically demonstrate that the serial correlation in prices is often negative and also show that our model predictions are consistent with this empirical regularity.

4.1 Multiple Modes

Our empirical examination of the IRI Marketing data set is consistent with the finding in Rao, Arjunji and Murthi (1995) that the distribution of prices is multi-modal. A detailed description of the data is available in Bronnenberg, Kruger and Mela (2008). The evidence presented here is from the year 2006 in the data set.\(^{20}\) We plotted the histogram of weekly prices for several SKU’s across all categories available in the dataset to

\(^{20}\) The data for 2006 was the most recent year available to us at the time of this analysis.
understand the shape of pricing distributions. We found that several SKU’s have small price variation and consequently have a single mode. Given our goal to understand the pricing patterns that occur due to promotions, we focus on SKU’s that have sufficient price variability in the weekly prices. Given this objective we choose the top fifteen SKU’s with the most price variation from each of the 28 categories (totaling 420 SKU’s). For the purpose of illustration in Figure 7 we provide the histograms from an SKU from three different categories: blades, coffee and soup from left to right respectively. The x-axis represents the price and the y-axis represents the frequency in percentage.

![Figure 7: Histograms of Weekly Prices](image)

To determine the number of modes in the pricing distribution, we count only distinct prices that are charged with more than 5% frequency. So, for example in Figure 7, the picture in the middle has only 2 modes: one around price 9 and another one around 7. A summary of the number of SKUs out of fifteen with 1, 2 or more than 2 modes in their pricing distribution in each category can be found in Table A3 in the appendix. Across all categories 8% of the SKUs have a single mode, 38% of the SKUs have two modes and 54% of the SKU's have more than two modes. Furthermore, of the SKUs with only two modes 71% of the SKUs have one internal mode. If we include SKUs with more than two modes then across all categories 81% of them have an internal mode. In conclusion, the finding that the pricing distribution is multi-modal and that there are interior modes is a prevalent phenomena.

Given the prevalence of the pricing distributions that are multi-modal with interior modes we now demonstrate that the equilibrium pricing distributions implied by our theory can explain these patterns. In our model, the unconditional distribution, which is the convex combination of state dependent distribution, will yield internal modes in the equilibrium pricing distribution because the lower bounds of the pricing distributions and
the upper bound of the hole are different across two states. For illustration, in Figure 8 we plot the unconditional equilibrium density of prices for the following set of parameter values: $r=100$, $n=2$ and $3$, $\nu=0.6$, $\lambda=0.6$ and $\delta=0.95$.

![Figure 8: Predicted Equilibrium Pricing Density](image)

It is worth noting that the equilibrium mixing density implied by our theory (Figure 8) is consistent with the multi-modal pricing patterns with interior modes often observed in practice. For a wide range of parameters the equilibrium density predicted by our model is multi-modal with interior modes.

### 4.2 Negative Serial Correlation in Prices

Although, the negative serial correlation in prices has been documented by earlier empirical studies (Blattberg and Neslin 1990, Pesendorfer 2002), in this section we provide additional evidence of negative serial correlation using the IRI Marketing dataset. First we examine the prevalence of negative serial correlation in prices by plotting the weekly prices of all 420 SKUs over the 52 week period. Visual inspection of these time series shows that there are no structural breaks or trends in the pricing patterns. Furthermore, the data suggests the possible presence of negative serial correlation. As an example we present the time series of SKUs from the same three categories: blades, coffee and soup from left to right respectively in Figure 9. The x-axis represents the week number and the y-axis represents the price.
Recall that in our model consumers enter the market in every period. Consequently, the inter-purchase time assumed in our model is a single period. To formally test for negative serial correlation in prices we use the purchase cycle data in the IRI data set as a proxy for a unit of time in our model. We estimate the random coefficient regression model at the SKU level for each category:  

$$ p_t = \alpha + \rho p_{t-1} + \epsilon_t, $$

where $t$ is week index and $l$ is the purchase cycle (lag) in weeks for that category. The parameter $\rho$ is the serial correlation in prices.

In Table A4 (see appendix) we report the mean serial correlation and its variance across the SKUs in each category. Although in several categories the mean estimates are positive, several SKUs in the category still exhibit negative serial correlation. For instance, in the peanut butter category the average serial correlation across SKUs is 0.07 and yet 8 of 15 SKUs in the category have a negative estimate for the serial correlation parameter. On the other hand, in the soup category the mean serial correlation is 0.27 and the variance across SKUs in the category is 0.03, which is rather small relative to the mean. All fifteen SKUs in this category exhibit positive serial correlation. An examination of the time series plot of SKUs in this category reveals that the frequency of promotions in this category is quite low and prices are relatively constant (please refer to the third plot in Figure 9 as an example) which explains the positive serial correlation.

We should also note that the mean estimate of the serial correlation is negative in many of the 28 product categories. For example, in the laundry detergent category the average serial correlation is -0.17 and in the frozen dinner category it is -0.19. Correspondingly, 13 out of 15 SKUs in the former and 12 out of 15 SKU’s in the latter category exhibit negative serial correlation. If we look at the individual estimates of all
420 SKU’s across all categories, approximately half of the SKUs exhibit negative serial correlation (see Table A4 in the appendix).

Interestingly, visual inspection of the histograms of prices and the time series plots across SKUs (Figures 7 and Figure 9) reveal that the SKUs with multiple modes tend to exhibit negative serial correlation in prices while SKUs that do not have multiple modes tend to exhibit positive serial correlation. We now provide the intuition for why our model predicts the negative serial correlation in prices.

In the presence of stockpiling, the state independent pricing distribution implied by our model generates a negative serial correlation in prices. Note from (10) that $F_1(p)$ first order stochastically dominates $F_0(p)$ for all $\lambda > 0$ in any equilibrium with stockpiling which implies that the expected price in $s=1$ is greater than the expected price in $s=0$. The first order stochastic dominance also implies that the probability that a firm will charge prices below any price, including the stockpiling threshold, in state $s=1$ is lower than that in state $s=0$, $F_1(t) < F_0(t) \forall \lambda > 0$. This means that when a firm is in state $s=0$ it offers deep discounts more often (relative to when it is in state $s=1$) and is therefore more likely to transition to state $s=1$ where expected prices are higher. Conversely, when a firm is in state $s=1$ it offers shallow discounts more often (relative to when it is in state $s=0$) and therefore more likely to transition to state $s=0$, where expected prices are lower. Thus, the equilibrium promotional strategy in our model generates negative serial correlation in prices.

To the best of our knowledge our model is the first to offer an explanation for both the presence of multiple modes in pricing distribution and the negative serial correlation of prices. In the next section, we extend our base model to incorporate the possibility of increased consumption when consumers have excess inventory at home and show that qualitative findings of the base model do not change.

5. **Extension: Stockpiling with Flexible Consumption**

In the base model we assume that the consumption rate is constant so that strategic consumers carry inventory forward in the event that they stockpile. In this extension we relax this assumption to allow for flexible consumption. We assume that
when strategic consumers have inventory, they may consume the additional unit with probability \( \theta \in (0, 1) \). Note that \( \theta = 1 \) corresponds to our base model with no flexible consumption while \( \theta = 1 \) corresponds to the case in which strategic consumers exhaust all inventory and are perpetually in a state of zero inventory at the time of purchase whether or not they bought an additional unit in the previous period. The demand dynamics in the flexible consumption case is described in Figure 10. The stockpiling threshold in this case is computed in a manner similar to the base case. It is determined by comparing the strategic consumers’ continuation utility from buying an extra unit for the future, but consuming it in the same period with probability \( \theta \) versus that from not stockpiling.

Figure 10: State-Space and Demand Dynamics with Flexible Consumption

21 If strategic consumers do not have any inventory they purchase at least one unit for current consumption and may stockpile if prices are lower than the stockpiling threshold. If they do not stockpile they transition to a state with no inventory in the next period. If they do stockpile then a fraction \( \theta \) consume both units and end up with no inventory in the next period while the remaining \( (1-\theta) \) consume only one unit and end up with an inventory of one unit. In this case, the market has a fraction \( \theta \) with no inventory and a fraction \( (1-\theta) \) with an inventory of one unit. In the event that strategic consumers do have inventory they do not purchase for current consumption if prices are high and transition to a state with no inventory in the next period. If firms offer a deep discount they stockpile and purchase one unit. Again a fraction \( \theta \) consume both units and end up with no inventory in the next period while the remaining \( (1-\theta) \) consume just one unit and end up with an inventory of one.
The solution procedure is very similar to our base model. Additional details can be found in the Appendix. In Proposition 2, we present the main finding from incorporating the possibility of increased consumption due to stockpiling.

**Proposition 2**: Firms' profits are independent of the probability of increased consumption due to stockpiling by strategic price sensitive consumers:

\[
V_0 = V_\theta = \frac{rV}{n(1 - \delta_F)}.
\]

Proof: See appendix.

Interestingly, the firms’ equilibrium profits do not depend on the probability of increased consumption due to stockpiling, \(\theta\), by the strategic price sensitive consumers. In fact, the expected profits with flexible consumption are identical to that obtained in our base model. This is surprising because firms’ profits are unaffected despite increased consumption by consumers. To see the intuition behind this result, consider the case when \(\theta=1\) so that all consumers who stockpile consume both units in the current period. In this case consumers are perpetually in state \(s=0\). As in the base model, competition for price sensitive consumers will result in firms’ mixing over a range of prices with the reservation price, \(r\), in the support of the mixing distribution. The equi-profit condition requires that the expected profit from charging \(r\) be the same as charging any other price in the support of the mixing distribution. The expected profit from charging \(r\) is independent of \(\theta\), which yields the above result. It is easy to extend this argument to the case when \(\theta \in (0,1)\). Intuitively all potential profits from price sensitive consumers are competed away in equilibrium; hence increased consumption due to stockpiling by strategic consumers has no impact on firms’ equilibrium profits.

Other findings from this setup are qualitatively similar to the base model. Recall that in our base model with no flexible consumption, firms compete relatively more intensely in the state in which strategic consumers do not have inventory and harvest in the subsequent period. With flexible consumption the force to harvest gets weaker as some of the strategic consumers reenter the market and purchase a unit even when firms offer a shallow discount and purchase two units when firms offer deep discounts. This is to be expected as \(s=\theta\) in this
extension can be viewed as a convex combination of \( s=0 \) and \( s=1 \) in the base model, where firms compete intensely in \( s=0 \) and harvest in \( s=1 \).

6. Concluding Remarks

In this paper, we developed a model that incorporates the dynamic effects of price promotions. Specifically, we allow for the category demand to expand in promotional periods by allowing for inter-temporal shifts. We do this by assuming that in the promotional periods when the firms offer deep discounts (below the stockpiling threshold) some consumers purchase not only for current consumption but also stockpile for future consumption. An inter-temporal shift in demand induces state-dependence such that firms’ pricing strategies in any given period depend not only on their strategy in the previous period but also on that of their competitors in the previous period. The demand dynamics lead to several interesting findings. First and most importantly, the equilibrium pricing distributions implied by our theory are consistent with the pricing distributions observed in practice. We find that the equilibrium mixing distribution has firms offering shallow discounts or deep discounts with the depth and frequency of these discounts depending on whether or not a deep discount was offered by any firm in the previous period (i.e. depends on the state). Second, endogenizing the stockpiling threshold allows us to highlight market conditions in which stockpiling may or may not occur in equilibrium. Under the market conditions in which stockpiling can occur in equilibrium the pricing distribution is multi-modal with interior modes. When market conditions are such that the equilibrium pricing strategies do not permit stockpiling behavior, the equilibrium pricing distribution is at most bi-modal with modes at the end of the support, which is consistent with the predictions from extant theory (Varian 1980, Narasimhan 1988).

Our analysis also demonstrates the importance of explicitly incorporating strategic behavior on the part of the consumer in determining the stockpiling threshold endogenously. Strategic price sensitive consumers purchase an extra unit when deep discounts are offered and if firms offer shallow discounts in the subsequent period they stay out of the market and consume from their inventory. This induces an interesting cycle in which firms compete intensely by offering deep discounts in one period and then
harvest in the following period by offering shallow discounts. The dependence on past pricing decisions is interesting in that even if only one firm offers a deep discount in the previous period, it is in the best interest of all the firms to charge higher average prices in the following period as the clientele mix is now skewed more towards price insensitive consumers. This is because a deep discount (below the stockpiling threshold), even if it is offered by only one firm serves to clear the demand from strategic price sensitive consumers that stockpile. The benefit of offering a deep discount in the following period to induce them to stockpile (purchase only for future consumption) is dominated by the cost of subsidizing the price insensitive consumers. Another benefit (to firms) from offering a deep discount is that by selling in advance to all price sensitive strategic consumers, competition for price sensitive consumers is mitigated in subsequent periods. Finally, we highlight the interplay of market characteristics with promotional strategies as it relates to the frequency and depth of shallow vs. deep discounts, which in turn offers many interesting insights for managers.

As a caveat, note that price insensitive consumers in our model are indistinguishable from consumers who are loyal or variety seeking but price insensitive. In this light our model can be viewed as one that examines a market where only some switchers stockpile. Interested readers may refer to Gangwar, Kumar and Rao (2012) who investigate what happens when loyal consumers engage in stockpiling.

The key findings from incorporating flexible consumption are qualitatively similar. Recall that in our base model with no flexible consumption, firms compete relatively more intensely in the state in which strategic consumers do not have inventory and harvest in the subsequent period. With flexible consumption, the force to harvest gets weaker as some of the strategic consumers reenter the market and purchase a unit even when firms offer a shallow discount and purchase two units when firms offer deep discounts. However, extra consumption due to stockpiling by strategic price sensitive consumers does not generate additional revenues for the firm.

References


Appendix

Table A1: Notation

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<tbody>
<tr>
<td>$s$</td>
<td>Subscript for state = 0 or 1</td>
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</tbody>
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**Exogenous**

<table>
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<th>$n$</th>
<th>Number of firms</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r$</td>
<td>Reservation price</td>
</tr>
<tr>
<td>$c$</td>
<td>Marginal cost</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Fraction of price insensitive consumers</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Fraction of strategic consumers</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Probability of consuming additional unit</td>
</tr>
<tr>
<td>$\delta_c$</td>
<td>Consumer discount factor</td>
</tr>
<tr>
<td>$\delta_F$</td>
<td>Firms’ discount factor</td>
</tr>
</tbody>
</table>

**Endogenous**

<table>
<thead>
<tr>
<th>$t$</th>
<th>Stockpiling threshold</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h_s$</td>
<td>Upper bound of the hole in state $s$</td>
</tr>
<tr>
<td>$l_s$</td>
<td>Lower bound of price in state $s$</td>
</tr>
<tr>
<td>$F_s$</td>
<td>Cumulative distribution function in state $s$ for prices</td>
</tr>
<tr>
<td>$U_s$</td>
<td>Consumer’s continuation utility in state $s$</td>
</tr>
<tr>
<td>$V_s$</td>
<td>Firms’ continuation payoff in state $s$</td>
</tr>
</tbody>
</table>

Figure A1: Weekly Time Series Plot of Prices
**Proof: Lemma 1**

We begin by examining the decision problem of forward-looking consumers in each state. In $s=0$, these consumers have no inventory for current period consumption. Consequently, they have one of two options: purchase only one unit for current period consumption and end up in the $s=0$ (with no inventory) next period or stockpile; purchase two units, consume one unit and end up in $s=1$ (with an inventory of one unit). Given the lowest price, $p_0^{\min}$, in the current period strategic consumers will engage in stockpiling only if the following inequality holds:

$$\min_0 \min_0 \left( 1 + \delta_c \right) r - 2p_0^{\min} + \delta_c U_1 \geq r - p_0^{\min} + \delta_c U_0$$

The left hand side of this inequality represents the utility from stockpiling and transitioning to $s=1$ in which case the consumers’ expected continuation utility is $U_1$. The right hand side of the inequality represents the utility from not stockpiling and transitioning to $s=0$ in which case the expected continuation utility is $U_0$. Stockpiling is the preferred option if:

$$p_0^{\min} \leq \delta_c r + \delta_c U_1 - \delta_c U_0.$$

In $s=1$, strategic consumers have an inventory of one unit. Given that they have inventory for current period consumption, the two options facing them in this state are: purchase nothing, consume the unit in inventory and end up in $s=0$ (with no inventory) or stockpile and purchase one unit and end up in $s=1$ (with an inventory of one unit) next period. Once again given the lowest price, $p_1^{\min}$, in the current period strategic consumers will engage in stockpiling iff:

$$\delta_c r - p_1^{\min} + \delta_c U_1 \geq \delta_c U_0 \text{ or } p_1^{\min} \leq \delta_c r + \delta_c U_1 - \delta_c U_0.$$

We define the **stockpiling threshold**, $t$, as the price where this inequality binds or $t = \delta_c r + \delta_c U_1 - \delta_c U_0$. Note that the stockpiling threshold, $t$, and expected continuation utilities $U_0$ and $U_1$ are all endogenously determined, and are the same in both states.

QED.
**Equilibrium Mixing Distributions**

We solve for symmetric equilibrium, which is also unique sub game perfect (Baye et. al. 1992). We use the same logic as Narasimhan (1988) to rule out mass points in the interior of the support. Intuitively, suppose there was a mass point in the interior of the support then all firms will charge that price with positive probability in equilibrium; in that case one firm can profitably deviate by lowering its price, which contradicts the equilibrium. Please refer to Narasimhan (1988) for a detailed proof.

Recall in (1) to (4), firm’s demand depends not only on the state but also on whether a firm charges a price above or below the stockpiling threshold. Under the assumption that if consumer sees a price equal to the threshold she stockpiles. We substitute \( p = t \), in (6) and (8) and use Proposition 1 to evaluate the probability of firm offering a price less than or equal to the stockpiling threshold through following equations:

\[
\begin{align*}
    t \left[ \frac{V}{n} + (1 - \nu)(1 + \lambda)(1 - F_0(t))^{n-1} \right] &= \frac{rv}{n(1 - \delta_x)} \\
    t \left[ \frac{V}{n} + (1 - \nu)(1 - F_1(t))^{n-1} \right] &= \frac{rv}{n(1 - \delta_x)}
\end{align*}
\]

Further algebraic manipulation of the above equations yields the firms’ probability of offering a price below the stockpiling threshold in each state:

\[
F_0(t) = 1 - \left[ \frac{v(r - t)}{nt(1 - \nu)(1 + \lambda)} \right]^{1/(n-1)}, \quad F_1(t) = 1 - \left[ \frac{v(r - t)}{nt(1 - \nu)} \right]^{1/(n-1)}
\]

To solve for the cumulative distribution functions above and below the stockpiling threshold in each state, we substitute values of \( V_s \) from Proposition 1 and \( F_s(t) \) from above equalities in equations (5) to (8). We get the following CDF:\(^\ast\):

\[
F_0^a(p) = 1 - \left[ \frac{v(r - p)}{np(1 - \nu)} \right]^{1/(n-1)} \quad \forall p \in (t, r) \\
F_0^b(p) = 1 - \left[ \frac{v(r - p)}{np(1 - \nu)(1 + \lambda)} \right]^{1/(n-1)} \quad \forall p \in \left[ l_0, t \right]
\]

(14)
\[ F_i^a(p) = 1 - \left[ \frac{v(r-p)}{np(1-v)(1-\lambda)} \right]^{\frac{1}{(a-1)}} \quad \forall p \in (t,r] \] 

\[ F_i^b(p) = 1 - \left[ \frac{v(r-p)}{np(1-v)} \right]^{\frac{1}{(b-1)}} \quad \forall p \in [l,t] \] 

(15)

where superscript ‘a’ (‘b’) denotes the price range above (below) the stockpiling threshold.

Given the functional form of CDF’s in (14) and (15) we evaluate the lower bounds of support in each state. We know that the CDF’s at the lower bounds in each state must evaluate to zero. Hence

\[ F_0^b(l_0) = 1 - \left[ \frac{v(r-l_0)}{nl_0(1-v)} \right]^{\frac{1}{(b-1)}} = 0 \quad \text{and} \quad F_1^b(l_1) = 1 - \left[ \frac{v(r-l_1)}{nl_1(1-v)} \right]^{\frac{1}{(b-1)}} = 0 \]

This leads to following lower bounds in each state:

\[ l_0 = \frac{rv}{v + n(1+\lambda)(1-v)} , \quad l_1 = \frac{rv}{v + n(1-v)} \]

**Proof: Lemma 2**

Using the equi-profit condition in (9) it is easy to see that \( F_i^a(t) < F_i^b(t) , \forall \lambda > 0 \), which violates the non-decreasing property of the cumulative distribution function. This contradiction can only be resolved if prices in the neighborhood of \( t \) are not in the support of the equilibrium mixing distribution. Suppose in an equilibrium the hole starts at some lower bound \( k \) that is below the stockpiling threshold, \( t \). Knowing that the continuation payoffs are same for all prices below \( t \), we focus on only current period payoffs. In such equilibrium, consider the focal firm’s deviation of shifting a little mass from below the hole to inside the hole. This deviation will result in higher profits for the focal firm because the firm can charge a higher price without affecting the probability of attracting price sensitive consumers and the stockpiling decision of strategic consumers. This profitable deviation violates the equilibrium condition. Hence, the hole must extend above the stockpiling threshold, \( t \). Equating the value of CDF above the stockpiling threshold at the upper bound of the hole, \( F_i^a(h_i) \), to the values of CDF below the
stockpiling threshold at \( t \), \( F_s^b(t) \), respectively in each state \( s=\{0, 1\} \) provides the following equalities:

\[
F_0^a(h_0) = F_0^b(t) \iff 1 - \left[ \frac{v(r-h_0)}{nh_0(1-v)} \right]^{1/(\alpha-1)} = 1 - \left[ \frac{v(r-t)}{nt(1-v)(1+\lambda)} \right]^{1/(\alpha-1)}
\]

\[
F_1^a(h_1) = F_1^b(t) \iff 1 - \left[ \frac{v(r-h_1)}{nh_1(1-v)(1-\lambda)} \right]^{1/(\alpha-1)} = 1 - \left[ \frac{v(r-t)}{nt(1-v)} \right]^{1/(\alpha-1)}
\]

from which we solve for the upper bounds of holes in each state:

\[
h_0 = \frac{rt(1+\lambda)}{r+t\lambda}, \quad h_1 = \frac{rt}{r+t\lambda-r\lambda}
\]

QED.

By appropriately defining the support of the equilibrium mixing distribution above and below the stockpiling threshold, \( F_s^a(p \mid t) \) and \( F_s^b(p \mid t) \) in (14) and (15) by substituting the lower bound of support, \( l_s \), and the upper bound of hole, \( h_s \), in each state \( s=\{0, 1\} \), we fully characterize the equilibrium mixing distribution in (10) when \( t \) is in the interior of the support.

**State Transition Probabilities**

We need to know the state transition probabilities to compute the unconditional CDF’s and pdf’s under different market conditions. Recall that although the stockpiling threshold is same across both states, the mixing probabilities are different. However, the state in any given period only depends on whether the lowest price in the previous period was above or below the stockpiling threshold. Therefore, if the lowest price in the previous period was below the stockpiling threshold, \( p_s^{\text{min}} \leq t \), the current period state will be \( s=1 \). Formally, \( \Pr(s=1) = \Pr(p_0^{\text{min}} \leq t) \Pr(s=0) + \Pr(p_1^{\text{min}} \leq t) \Pr(s=1) \)

Substitute the appropriate state dependent conditional probabilities for minimum prices, \( \Pr(p_0^{\text{min}} \leq t) = 1 - (1 - F_0(t))^n \) and \( \Pr(p_1^{\text{min}} \leq t) = 1 - (1 - F_1(t))^n \) in the forgoing equation to solve for unconditional state probability:

\[
\Pr(s=0) = \frac{(1 - F_1(t))^n}{1 - (1 - F_0(t))^n + (1 - F_1(t))^n} = (1 - \Pr(s=1))
\]
Consumer Utilities

Table A2: First derivatives of discounted utilities with respect to market parameters

<table>
<thead>
<tr>
<th>( \frac{d(\text{row})}{d(\text{column})} )</th>
<th>Consumer discount factor, ( \delta_c )</th>
<th>Number of firms, ( n )</th>
<th>Proportion of stockpiling consumers, ( \lambda )</th>
<th>Proportion of price insensitive consumers, ( \nu )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strategic consumer’s expected utility, ( U )</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>Strategic consumer’s expected utility in state 0, ( U_0 )</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>Difference of strategic consumer’s expected utility in state 0 and 1, ( U_0 - U_1 )</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>Strategic consumer’s expected utility in state 1, ( U_1 )</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>-</td>
</tr>
</tbody>
</table>

Flexible Consumption

Unique Stockpiling Threshold

Similar to our base model, we redefine two states as state \( s=0 \) and state \( s=\theta \). However unlike the base case, in this case states do not directly correspond to strategic consumers’ inventory position. In state \( s=\theta \), \( \theta \) proportion of strategic consumers have no inventory (with continuation utility of \( U_{\theta_0} \)) and \((1-\theta)\) fraction have inventory of one unit (with continuation utility of \( U_{\theta_1} \)).

In state \( s=0 \), when the strategic consumer has no inventory he will buy two units and stockpile only if the following inequality holds.

\[
\theta \left[ 2r - 2p_0^{\min} + \delta_c U_{\theta_0} \right] + (1-\theta) \left[ (1+\delta_c) r - 2p_0^{\min} + \delta_c U_{\theta_1} \right] \geq r - p_0^{\min} + \delta_c U_0
\]

The LHS and RHS represent the utility, at the time of purchase, from stockpiling and not stockpiling respectively. The first term on the LHS represents the utility from buying and consuming two units which happens with the probability \( \theta \) and the second term on the
LHS is the utility from buying two units but consuming only one which happens with the complementary probability \((1 - \theta)\). The RHS is the utility from buying only one unit (not stockpiling) and staying in state \(s = 0\). The strategic consumer in state \(s = 0\) will stockpile only if the minimum price in the market, \(p_{0}^{\text{min}}\), is below, \(t\):

\[
p_{0}^{\text{min}} \leq t = \left( \theta + (1 - \theta)\delta_c \right) r + \delta_c \left( \theta U_{\theta_0} + (1 - \theta)U_{\theta_1} - U_0 \right).
\]  

(16)

Analogous comparison of the utilities for the strategic consumer with zero inventory in state \(s = \theta\) provides exactly the same condition for the stockpiling decision as mentioned above.

On the other hand, utility comparison for consumers with inventory of 1 in state \(s = \theta\) yields the following inequality, \(\theta \left( r - p_{\theta}^{\text{min}} + \delta_c U_{\theta_0} \right) + (1 - \theta) \left( \delta_c r - p_{\theta}^{\text{min}} + \delta_c U_{\theta_1} \right) > \delta_c U_{\theta_0}\).

Notice that the RHS of the above inequality does not includes the utility from the current period consumption, which is already accounted for in the last period’s purchase decision. However, the strategic consumer’s stockpiling threshold is the same as \((16)\):

\[
t = \left( \theta + (1 - \theta)\delta_c \right) r + \delta_c \left( \theta U_{\theta_0} + (1 - \theta)U_{\theta_1} - U_0 \right).
\]

**Proof: Proposition 2**

Given the demand dynamics described in Figure 10, we can write the equi-profit conditions in both state \(s = \{0, \theta\}\). In particular, all prices in the support of the equilibrium mixing distribution must provide the same continuation profit. In state \(s = 0\) this leads to,

\[
p \left[ \frac{V}{n} + (1 - \nu)(1 - F_{\theta}(p))^{n-1} \right] + \delta_F \left[ V_0 (1 - F_{\theta}(t))^{n-1} + V_{\theta} (1 - (1 - F_{\theta}(t))^{n-1}) \right] = V_0 \quad \forall p \in (t, r)
\]

\[
p \left[ \frac{V}{n} + (1 - \nu)(1 + \lambda)(1 - F_{\theta}(p))^{n-1} \right] + \delta_F V_{\theta} = V_0 \quad \forall p \in [l_{\theta}, t]
\]

In state \(s = \theta\) the equi-profit conditions are

\[
p \left[ \frac{V}{n} + (1 - \nu)(1 - \lambda + \lambda \theta)(1 - F_{\theta}(p))^{n-1} \right] + \delta_F \left[ V_0 (1 - F_{\theta}(t))^{n-1} + V_{\theta} (1 - (1 - F_{\theta}(t))^{n-1}) \right] = V_{\theta} \quad \forall p \in (t, r)
\]
\[ p\left[ \frac{V}{n} + (1-v)(1+\lambda\theta)(1-F_\theta(p))^{n-1} \right] + \delta V = V_\theta \quad \forall p \in [l_\theta, t] \]

Notice that these equi-profit conditions are similar to (5) to (8) outlined in the main paper. Similar to Proposition 1, simultaneously solving the above equi-profit conditions at the reservation price \( p=r \) in both states \( s=\left\{ 0, \theta \right\} \) directly leads to following equality: \( V_0 = V_\theta = \frac{rv}{n(1-\delta_\theta)} \)

**QED**

The solution procedure to identify the equilibrium pricing distribution in the flexible consumption case is similar to the base case. For any given stockpiling threshold the equilibrium distributions for the flexible consumption case in both states are:

\[
F_s(p,t) = \begin{cases} 
1 & , \quad r \leq p \\
1 - \left[ \frac{v(r-p)_{npt}}{npt(1-v)(1+\lambda)} \right]^{n-1} & , \quad t \leq p < \frac{r(1+\lambda)}{r+\lambda} \\
1 - \left[ \frac{v(r-t)_{npt}}{npt(1-v)(1+\lambda)} \right]^{n-1} & , \quad \frac{rv}{v+n(\lambda+1)(1-v)} \leq p < t \\
0 & , \quad p \leq \frac{rv}{v+n(\lambda+1)(1-v)} 
\end{cases}
\]

Given the equilibrium pricing distributions we can compute strategic consumers' net discounted utilities in each state. The following equations are analogous to (11) and (12) of the base case.

\[
U_{\theta_0} = (1-F_\theta(t))^n \left( r - E_{p_{\theta_0}^{\min}} \left[ p_{\theta_0}^{\min} \right] + \delta U_0 \right) \\
+ \left(1-(1-F_\theta(t))^n\right) \left[ (1+\theta+(1-\theta)\delta_c) r - 2E_{p_{\theta_0}^{\min}} \left[ p_{\theta_0}^{\min} \right] + \theta\delta U_{\theta_0} + (1-\theta)\delta U_{\theta_1} \right] \\
U_{\theta_1} = (1-F_\theta(t))^n \delta_c U_0 \\
+ \left(1-(1-F_\theta(t))^n\right) \left[ (1+\theta+(1-\theta)\delta_c) r - E_{p_{\theta_1}^{\min}} \left[ p_{\theta_1}^{\min} \right] + \theta\delta_c U_{\theta_0} + (1-\theta)\delta_c U_{\theta_1} \right]
\]
\[ U_0 = (1 - F_0(t))^\rho \left( r - E_{t < p_0^{\min}}[p_0^{\min}] + \delta_c U_0 \right) \]
\[ + \left( 1 - (1 - F_0(t))^\rho \right) \left( (1 + \theta + (1 - \theta)\delta_c) r - 2 E_{p_0^{\min} \leq \bar{p}}[p_0^{\min}] + \theta \delta \bar{U}_0 + (1 - \theta)\delta \bar{U}_1 \right) \]

We obtain \( \bar{U}_0, \bar{U}_1 \) and \( U_0 \) as a function of the stockpiling threshold and firms pricing strategy by simultaneously solving the foregoing equations. The endogenous stockpiling threshold \( t^* \) in the flexible consumption case is the solution of the following equality that is obtained by substituting \( \bar{U}_0, \bar{U}_1 \) and \( U_0 \) in (16):

\[ t^* = \left( \theta + (1 - \theta)\delta_c \right) r + \delta_c \left( \theta \bar{U}_0(t^*) + (1 - \theta)U_1(t^*) - U_0(t^*) \right) \]
Empirical Regularities in IRI Marketing Dataset

The number in each row in Table A3 represents the number of SKUs in the category with 1, 2 and more than 2 modes in the pricing distribution of fifteen SKU’s with most price variation in each of the 28 categories in the IRI marketing 2006 dataset.

Table A3: Number of Modes

<table>
<thead>
<tr>
<th>Category</th>
<th>Number of Modes above 5%</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>Beer</td>
<td>1</td>
</tr>
<tr>
<td>Blades</td>
<td>0</td>
</tr>
<tr>
<td>Coffee</td>
<td>0</td>
</tr>
<tr>
<td>Cold cereal</td>
<td>1</td>
</tr>
<tr>
<td>Deodorant</td>
<td>2</td>
</tr>
<tr>
<td>Diapers</td>
<td>4</td>
</tr>
<tr>
<td>Facial tissue</td>
<td>0</td>
</tr>
<tr>
<td>Frozen dinners/entrees</td>
<td>0</td>
</tr>
<tr>
<td>Frozen pizza</td>
<td>0</td>
</tr>
<tr>
<td>Household cleaner</td>
<td>1</td>
</tr>
<tr>
<td>Laundry detergent</td>
<td>1</td>
</tr>
<tr>
<td>Margarine/spreads/butter</td>
<td>3</td>
</tr>
<tr>
<td>Mayonnaise</td>
<td>3</td>
</tr>
<tr>
<td>Milk</td>
<td>2</td>
</tr>
<tr>
<td>Mustard ketchup</td>
<td>1</td>
</tr>
<tr>
<td>Paper towels</td>
<td>2</td>
</tr>
<tr>
<td>Peanut butter</td>
<td>0</td>
</tr>
<tr>
<td>Photography supplies</td>
<td>1</td>
</tr>
<tr>
<td>Razors</td>
<td>0</td>
</tr>
<tr>
<td>Salty snacks</td>
<td>2</td>
</tr>
<tr>
<td>Shampoo</td>
<td>0</td>
</tr>
<tr>
<td>Soup</td>
<td>6</td>
</tr>
<tr>
<td>Spaghetti/Italian sauce</td>
<td>0</td>
</tr>
<tr>
<td>Sugar substitutes</td>
<td>2</td>
</tr>
<tr>
<td>Toilet tissue</td>
<td>1</td>
</tr>
<tr>
<td>Toothbrush</td>
<td>0</td>
</tr>
<tr>
<td>Toothpaste</td>
<td>1</td>
</tr>
<tr>
<td>Yogurt</td>
<td>0</td>
</tr>
<tr>
<td>Total Percentages</td>
<td>8%</td>
</tr>
</tbody>
</table>

To test for the negative serial correlation in prices we use the purchase cycle data in the IRI data set as a proxy for a unit of time in our model. We estimate the random coefficient regression model at the SKU level for each category: \( p_t = \alpha + \rho p_{t-1} + \varepsilon \), where \( t \) is week index and \( l \) is the purchase cycle (lag) in weeks for that category. In Table A4
we report the mean serial correlation and its variance (across the SKUs) in each category. The last column in Table A4 denotes the number of SKUs out of 15 in each category in which the serial correlation estimates are negative.

<table>
<thead>
<tr>
<th>Category</th>
<th>Purchase Cycle</th>
<th>Lag in weeks</th>
<th>Average Serial Correlation</th>
<th>Variance of Serial Correlation</th>
<th># of SKUs (out of 15) with negative serial correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beer</td>
<td>67</td>
<td>10</td>
<td>0.10</td>
<td>0.04</td>
<td>6</td>
</tr>
<tr>
<td>Blades</td>
<td>106</td>
<td>15</td>
<td>-0.07</td>
<td>0.04</td>
<td>8</td>
</tr>
<tr>
<td>Coffee</td>
<td>65</td>
<td>9</td>
<td>-0.02</td>
<td>0.01</td>
<td>10</td>
</tr>
<tr>
<td>Cold cereal</td>
<td>48</td>
<td>7</td>
<td>-0.08</td>
<td>0.02</td>
<td>11</td>
</tr>
<tr>
<td>Deodorant</td>
<td>94</td>
<td>13</td>
<td>0.05</td>
<td>0.05</td>
<td>6</td>
</tr>
<tr>
<td>Diapers</td>
<td>55</td>
<td>8</td>
<td>0.15</td>
<td>0.02</td>
<td>3</td>
</tr>
<tr>
<td>Facial tissue</td>
<td>70</td>
<td>10</td>
<td>-0.06</td>
<td>0.04</td>
<td>9</td>
</tr>
<tr>
<td>Frozen dinners/entrees</td>
<td>51</td>
<td>7</td>
<td>-0.19</td>
<td>0.06</td>
<td>12</td>
</tr>
<tr>
<td>Frozen pizza</td>
<td>64</td>
<td>9</td>
<td>0.05</td>
<td>0.04</td>
<td>6</td>
</tr>
<tr>
<td>Household cleaner</td>
<td>82</td>
<td>12</td>
<td>0.17</td>
<td>0.05</td>
<td>4</td>
</tr>
<tr>
<td>Laundry detergent</td>
<td>80</td>
<td>11</td>
<td>-0.17</td>
<td>0.04</td>
<td>13</td>
</tr>
<tr>
<td>Margarine/spreads/butter</td>
<td>65</td>
<td>9</td>
<td>0.17</td>
<td>0.04</td>
<td>3</td>
</tr>
<tr>
<td>Mayonnaise</td>
<td>95</td>
<td>14</td>
<td>-0.11</td>
<td>0.03</td>
<td>12</td>
</tr>
<tr>
<td>Milk</td>
<td>29</td>
<td>4</td>
<td>0.19</td>
<td>0.03</td>
<td>3</td>
</tr>
<tr>
<td>Mustard ketchup</td>
<td>91</td>
<td>13</td>
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