Aggregate Risk, Bank Competition and Regulation in General Equilibrium

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Abstract

We develop a general equilibrium model of competitive banks to examine the optimal design of bank regulation. There is a continuum of equilibria of the unregulated economy that feature varying relative sizes of the financial and real sectors. The unregulated economy underinvests (overinvests) in production when aggregate risk is below (above) a threshold. An efficient allocation is implemented by a range of regulatory policies comprising of capital and liquidity requirements, deposit insurance, and bailouts financed by taxes, but there is a unique regulated equilibrium for a given regulatory policy. Capital and liquidity requirements move in opposing directions; an optimal regulatory policy that features a stricter capital requirement has a looser liquidity requirement. When aggregate risk is low, the efficient allocation can be implemented via deposit insurance and bailouts, but capital and liquidity requirements are necessary to ensure a unique regulated equilibrium. When aggregate risk is high, however, all four regulatory tools are essential components of an optimal regulatory policy. Capital and liquidity requirements that implement efficient regulatory policies do not vary with aggregate risk when it is below a threshold, but become tighter as aggregate risk increases above the threshold. When aggregate risk is low, it is efficient to subsidize depositors via taxes on entrepreneurs. At high levels of aggregate risk, however, it is efficient to provide subsidies to entrepreneurs finance by taxes on bank depositors and shareholders.

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1 Introduction

The 2007-2008 financial crisis was precipitated by the presence of insufficient liquid reserves and excessive debt levels in the financial system that made banks vulnerable to large aggregate negative shocks. Proponents of stricter bank regulation argue that higher capital requirements for banks reduce their insolvency risk. Opponents argue that higher capital requirements force banks towards costlier equity financing, and hamper banks' key role in transforming liquid debt securities into less liquid assets. Commentators have also highlighted the importance of imposing adequate liquidity reserve requirements on banks' assets, especially in the presence of aggregate risk. The debate over bank regulation remains unresolved as exemplified by the fact that the current U.S. government is aggressively undoing the moves towards stricter bank oversight embodied in the 2010 Dodd-Frank Act and the Basel III agreement. An evaluation of the effectiveness of regulation necessitates a unified framework that endogenizes the costs of bank debt and equity, and models the interaction between banks' assets and liabilities in the presence of aggregate risk.

We develop a novel and tractable general equilibrium model of competitive banks to investigate the optimal design of bank regulation. There is a continuum of equilibria of the unregulated economy that feature varying bank capital structures and relative sizes of the financial and real sectors of the economy. The unregulated economy underinvests (overinvests) in production when aggregate risk is below (above) a threshold. We show how regulatory intervention tools—|capital and liquidity requirements, deposit insurance and bank bailouts financed by taxation—|implement the efficient allocations in a decentralized economy. A given efficient allocation is, in fact, implemented by a range of regulatory policies that differ in the tightness of the capital requirement and the resulting size of the financial sector. Regulatory tools must be finely tuned to each other; an optimal regulatory policy that features a stricter capital requirement has a looser liquidity requirement. When aggregate risk is low, the efficient allocation can be implemented via deposit insurance and bank bailouts alone, but capital and liquidity requirements are necessary to ensure that the efficient allocation is uniquely implemented. When aggregate risk is high, however, all four regulatory tools are essential components of an optimal regulatory policy. Taken together, our results provide qualified support for proponents and opponents of stricter banking regulation. Lower capital requirements for banks could be optimal, but they must be accompanied by stricter liquidity requirements. We also obtain insights into how aggregate risk influences capital and liquidity requirements as well as the efficiency of depositor subsidies.

We consider an economy with a single consumption/capital good and continuums of households, banks and entrepreneurs. All agents have access to a safe, liquid asset that is in perfectly elastic supply. Capital is initially held by the ex ante identical households that include risk-averse depositors, who invest in bank deposits, and risk-neutral equity holders, who invest in bank and firm equity. Banks lend to risk-neutral entrepreneurs who operate firms with risky, concave technologies. Firms have identically distributed, but correlated binomial payoffs, and obtain capital via bank loans or equity issuance. Capital and bank loan markets are competitive.

There is a mass of "sectors" that are ex ante identical with depositors, banks and firms being distributed equally among the sectors. Each depositor invests her capital with a single bank, while an equityholder can supply capital to any bank or firm. Firms within a sector are exposed to sectoral shocks; a positive (negative) sectoral shock leads to a larger (smaller) proportion of firms in the sector simultaneously succeeding with the remaining firms failing. Banks are fully diversified within their sectors. As banks are ex ante identical, the firms in a bank's loan pool inherit the correlation structure of firms in the sector. The economy is also exposed to aggregate risk so the shocks to different sectors are correlated. Specifically, a proportion of sectors is exposed to an aggregate shock so that all these sectors simultaneously experience positive (negative) shocks depending on whether the realization of the aggregate shock is positive (negative). The shocks to the remaining sectors are independent. Agents only know the proportion of sectors that are exposed to the aggregate shock.

We show that there is a continuum of unregulated equilibria. Equilibria vary from one in which banks are financed purely with debt, and the real economy is financed with bank debt and firm equity, to a "full intermediation" equilibrium in which banks raise all available equity capital, and the real economy is financed entirely via bank debt. In other words, in the absence of any regulation, we could have equilibria that vary in the fragility of the financial sector. An equilibrium with greater bank equity capital also features greater bank debt (and, therefore, bank size), lower expected returns on bank deposits and equity, and higher production. The intuition for the results hinges on the fact that equity holders' capital is invested in firms either directly or indirectly via banks. Hence, the total amount of capital invested in firms (and, therefore, total production) is determined by the total deposit capital raised by banks. Further, the expected bank loan return equals the marginal return from production in a competitive loan market that is, in turn, determined by the total capital invested in firms. Hence, a competitive equilibrium is fully determined by the total deposit capital raised by banks. Because an increase in the total deposit capital increases the total capital invested in firms, an equilibrium with higher total deposit capital also features higher production. The concavity of firms' production technology implies that an increase in the total capital invested in firms is associated with a lower marginal return on production and, therefore, lower expected returns on bank loans, deposits and equity. To support an equilibrium with greater total deposit capital and a lower risk premium on bank deposits, banks' equity capital buffer must also be higher to lower the risk of bank deposits.

The properties of the unregulated equilibria highlight the key roles played by depositor risk aversion and strictly concave production technologies in our general equilibrium framework. Depositors invest in bank debt not because it is more liquid than other claims (they have access to a safe, liquid asset), but due to the endogenous premium that banks are able to provide risk-averse depositors by investing their capital in value-enhancing production. The endogenous cost of bank debt stems from the utility loss to risk-averse depositors due to the risk of bank insolvency. The debt cost benefit trade-off leads to a range of autarkic equilibria with nontrivial bank capital structures even in the absence of traditional frictions that lead to nonzero leverage. As firms' technology is strictly concave, bank size is pinned down in each unregulated equilibrium, and unregulated equilibria differ in the relative sizes of the financial and real sectors. In contrast, in models with linear technologies, bank size is indeterminate, and the relative sizes of the financial and real sectors are immaterial.

The inefficiencies in the unregulated economy stem from (i) limited market participation of depositors who cannot directly invest in firms, but only via banks (see Guiso et al. (2002)); (ii) imperfect diversification of depositors and banks; and (iii) incomplete markets due to which agents do not internalize the effects of aggregate risk on banks. We derive the efficient (or "first best") allocations by solving the planning problem that maximizes the total expected utility of depositors, while ensuring that entrepreneurs and equityholders receive their expected payoffs in the unregulated equilibria. For each unregulated equilibrium, therefore, there is a corresponding efficient allocation that maximizes the depositors' expected utility, while maintaining the other agents' payoffs at their unregulated equilibrium levels. The social planner always insures depositors against sectoral risk, but faces a trade-off between insuring depositors against aggregate risk and investing in production. If aggregate risk is below a threshold, the efficient allocation features full insurance for depositors. The planner chooses the investment level to maximize the expected payoff from production, and invests the remaining capital in the safe asset to ensure that depositors are fully insured. If aggregate risk is above a threshold, however, it is socially costly to fully insure depositors against aggregate risk. An increase in the aggregate risk above the threshold lowers investment in production and increases investment in the safe asset to offset the effects of higher aggregate risk on depositors' expected utility. Relative to the efficient allocation, the unregulated economy under-invests in production when aggregate risk is below a threshold, but over-invests when aggregate risk is above the threshold. As highlighted above, the presence of depositor risk aversion and a strictly concave technology implies that the planner trades off investment in production against sharing aggregate risk among depositors, equityholders and entrepreneurs. In contrast, in models with risk-neutral agents and linear technologies, the social objective is to simply increase expected production.

We examine the implementability of the efficient allocations with tools that are salient in banking regulation: capital and liquidity requirements, deposit insurance and taxpayer-funded bank bailouts. An efficient allocation can be implemented by a range of regulatory policies with the corresponding regulated equilibria differing in the tightness of the capital requirement and the resulting size of banks. Importantly, however, the different regulatory tools must be tuned to each other; a stricter capital requirement is associated with a looser liquidity requirement and less deposit insurance. In contrast with the multiplicity of unregulated equilibria, the equilibrium of the regulated economy is unique for a given set of regulatory tools so the efficient allocation is uniquely implemented by the regulatory policy.

An efficient allocation is determined by the total investments in firms and the safe asset. It can be implemented by (i) inducing depositors to invest entirely in banks via deposit insurance and (ii) imposing capital and liquidity requirements, which ensure that banks raise a certain level of equity and the total investments in the safe liquid asset and firms are consistent with the efficient allocation. Because the total equity capital is eventually invested in firms either directly or indirectly via banks, the efficient allocation can be implemented by multiple regulated equilibria that correspond to different levels of equity capital raised by banks. The liquidity requirement ensures that, depending on the total capital that banks raise in a regulated equilibrium, the total investment in the safe asset is efficient. Further, given a liquidity requirement, the capital requirement ensures that banks raise the right amount of equity, while the deposit insurance and bailout policy guarantee that the payoffs of depositors, equity holders and entrepreneurs are efficient. A stricter capital requirement implies that banks raise more equity so less capital is invested directly in firms by equity holders. Therefore, to ensure that the economy invests efficiently in production, the liquidity requirement on banks must be more lax. Further, greater equity capital implies that banks are less risky so less deposit insurance is required.

If aggregate risk is below the threshold that ensures the efficiency of full depositor insurance, the efficient allocation can be implemented via deposit insurance and bailouts. However, adding a capital and liquidity requirement is necessary to ensure that the regulated equilibrium is unique. Deposit insurance lowers the cost of risk faced by risk-averse depositors in the unregulated economy, while also ensuring that more deposit capital is invested in production via banks, thereby mitigating underinvestment. Because capital and liquidity requirements are only required to ensure equilibrium uniqueness, they do not vary with aggregate risk.

If aggregate risk is above the threshold which ensures that depositors must bear some aggregate risk, all four regulatory tools are necessary to implement the efficient allocation. Capital and liquidity requirements work in tandem to mitigate overinvestment in the unregulated economy, while deposit insurance and bailouts facilitate optimal risk-sharing among agents. Because the efficient allocation features increasing investment in the safe asset as aggregate risk increases in this region, the interval of possible values of the liquidity requirement| that can implement the efficient allocation shifts to the right. Further, for a given liquidity requirement that can implement the efficient allocation, the capital requirement becomes stricter as aggregate risk increases because the total amount of capital invested in the safe asset must increase so banks must raise more equity capital.

We also investigate the efficiency of the policy of subsidizing bank deposits. When aggregate risk is below a threshold, it is efficient to subsidize depositors by levying taxes (in expectation) on entrepreneurs. When aggregate risk is above the threshold, however, it is efficient to provide subsidies to entrepreneurs and tax depositors in expectation. The intuition hinges on the fact that the unregulated economy features underinvestment (overinvestment) relative to the efficient allocation when aggregate risk is low (high). When aggregate risk is low (high), the regulator, therefore, increases (decreases) investment in firms relative to the unregulated economy so the expected payoff from production is higher (lower). In the absence of any taxes or subsidies, therefore, the total payoff of entrepreneurs would be greater (less) than their total payoff in the unregulated equilibrium when aggregate risk is low (high). Because the efficient allocation maximizes the expected utility of depositors, while maintaing the payoffs of entrepreneurs and equityholders at their unregulated equilibrium levels, there must be an effective transfer in expectation from equity holders and entrepreneurs to depositors when aggregate risk is low, and vice versa when aggregate risk is high. Our results inform the debate generated by proposals to reduce, or even eliminate, the tax deductibility of bank debt interest payments to lower banks' incentives to take on excessive leverage. We argue that it is optimal to subsidize bank deposits when aggregate risk is low, but tax depositors (in expectation) when aggregate risk is high. General equilibrium effects and the trade-off between maximizing expected production, while achieving optimal risk sharing among depositors, equityholders and entrepreneurs via deposit insurance and bailouts financed by taxation play central roles in generating our results.

Our results show that there is, indeed, a range of efficient regulatory policies that feature relatively strict to relatively loose capital requirements, but different regulatory tools must be finely tuned to each other A policy with a strict capital requirement must also have a loose liquidity requirement to ensure that the economy invests optimally in value-enhancing production and safe, liquid assets that serve to make the banking system less vulnerable to aggregate risk. By channeling depositors' capital to the real economy, "banks" in our framework embody the roles of not just traditional banks, but also \shadow banks" that also perform a similar intermediation role. Our results, therefore, emphasize the importance of regulating the traditional and shadow banking sectors within a unified framework.

2 Related Literature

A large literature studies the design and impact of bank regulation from various perspectives (e.g., see Acharya et al. (2010), Hanson, Kashyap and Stein (2011) and Thakor (2014) for surveys). One stream of literature stresses the role of bank capital in inducing banks' monitoring efforts (Diamond (1984), Giammarino, Lewis, and Sappington (1993), Holmstrom and Tirole (1997), Allen, Carletti, and Marquez (2011), Mehran and Thakor (2011), Acharya, Mehran, and Thakor (2015)). Another strand of literature considers banks' liquidity provision role in accepting demand deposits and the role of deposit insurance in mitigating bank runs (Bryant (1980), Diamond and Dybvig (1983) , Gorton and Winton (1995), Diamond and Rajan (2000), Goldstein and Pauzner (2005)). A third strand of the literature highlights the role of capital requirements in mitigating debt overhang and asset substitution (e.g., Santos (2001)).

The aforementioned studies examine partial equilibrium frameworks that focus primarily on either the asset- or liability-side of banks' balance sheets and examine the effects of specific regulatory tools.

Further, the studies typically take the costs of bank equity and debt as exogenous. We build a unified, general equilibrium framework that endogenizes the costs of bank equity and debt, and incorporates the interactions between banks' assets and liabilities. In our model, banks are financial intermediaries that channel depositor capital to productive assets. Further, our model incorporates non-financial firms who receive financing from banks, and also compete with banks for equity capital. Capital and liquidity requirements as well as deposit insurance balance the trade-off between achieving optimal risk-sharing among risk-averse depositors, risk-neutral equity holders and entrepreneurs, while also ensuring that capital is invested in risky, but value-enhancing production. The competition between firms and banks for equity capital generates the link between capital and liquidity requirements; an optimal regulatory policy that features a tighter capital requirement must also have a looser liquidity requirement.

Gorton and Winton (1995, 2000) employ general equilibrium frameworks to show that, in the presence of investor demand for liquid securities, significantly higher bank capital requirements entail substantial social costs. The studies abstract away from liquidity requirements. Allen and Gale (2004) embed the Diamond-Dybvig (1983) model in a general equilibrium setting with aggregate shocks. They show that, when markets for aggregate risks are incomplete, there may be a role for regulation of liquidity provision. Gale and Ozgur (2005) adapt the Allen-Gale (2004) model to show that a regulatory policy that imposes a minimum capital requirement on banks may be sub-optimal, while Gale (2010) shows that increasing capital requirements may not necessarily lower the level of risk in the banking system. Gorton and Huang (2004) show that Government provision of liquidity in bank bailouts may be optimal when the market prices of distressed assets, which are determined in general equilibrium, depend on the available supply of liquidity. Morrison and White (2005) develop a model with adverse selection and moral hazard. They focus on the effects of capital requirements and demonstrate that they combat moral hazard when the regulator has a strong screening reputation, but otherwise substitute for screening ability. Boyd and De Nicolo (2005) show that, when competition among banks in asset and deposit markets is considered, increased bank competition may elevate (or lower) the risk of banks' portfolio if banks invest directly (or indirectly through a competitive loan market) in risky projects. Repullo and Suarez (2013) develop a general equilibrium model with risk-neutral agents and an exogenous excess cost of bank equity. They focus on comparing various bank capital regulation regimes, while abstracting away from other aspects of bank regulation such as liquidity requirements. DeAngelo and Stulz (2015) show that investor demand for liquidity causes bank debt to endogenously command a liquidity premium so that high bank leverage arises in equilibria even without taxes and other traditional motives for banks to choose high leverage. As in Gorton and Winton (1995, 2000), the social cost of higher capital requirements stems from a reduction in the supply of liquidity. Allen, Carletti and Marquez (2015) develop a general equilibrium model with risk-neutral agents in which the social cost of bank debt stems from exogenous bankruptcy costs, while the benefit arises due to banks' role as financial intermediaries who channel depositor capital to productive assets. Allen et al (2015) focus on the effects of deposit insurance and capital requirements, and only study the efficiency properties of the equilibria of their simple model without non-financial firms.[1]

We complement the above studies by showing how the policy tools that are salient in bank regulation| capital and liquidity requirements, deposit insurance and bailouts financed by taxes|combine to implement efficient allocations in a general equilibrium economy with financial and non-financial firms. Further, while the above studies largely assume universal risk-neutrality, the risk aversion of depositors plays a central role in our model and results. As we discussed in Section 1, the trade-off between the benefit and cost of bank debt arising due to depositor risk aversion leads to a range of unregulated equilibria with nontrivial bank capital structures even in the absence of traditional frictions that lead to nonzero bank leverage. Further, our incorporation of aggregate risk and a general, concave production technology generates a trade-off between providing insurance to risk-averse depositors against aggregate shocks, and investing in socially beneficial production. In contrast, many of the aforementioned studies assume linear technologies so that the sizes of the financial and real sectors are inconsequential, and the social objective is simply to maximize expected production.

Our general equilibrium analysis also generates the novel insights that there is, in fact, a range of efficient regulatory policies, but tighter capital requirements must be accompanied by looser liquidity requirements. Further, we derive implications for the impact of aggregate risk on optimal regulatory policies that are new to our study (to the best of our knowledge). The range of capital and liquidity requirements that form elements of optimal regulatory policies are unaffected by aggregate risk when it is below a threshold, but move to the right (become tighter) as aggregate risk increases above the

[1] Moreover, we characterize the set of Pareto efficient allocations via a traditional social planner's problem, who determines the investment and payoff allocations that maximize depositors' expected utility subject to risk-neutral equity holders and entrepreneurs receiving at least their unregulated equilibrium payoffs. In contrast, Allen et al (2015) consider a \hybrid" planning problem where the social planner maximizes depositors' expected utility, but subject to the the deposit rate being determined via clearing of bank deposit markets. Hence, it is not a priori clear that the allocation they characterize is Pareto efficient.

threshold. Depositor subsidies via deposit insurance and debt tax shields are efficient when aggregate risk is low, but inefficient when it is high.

3 The Model

Our model has two periods with a single consumption/capital good. We first consider an economy with unregulated banks, and then introduce regulation in Section 6. The economy has a continuum of mass 1 of financial intermediaries (banks); a continuum of mass $\mathcal{F}$ of entrepreneurs (firms) endowed with risky projects; a continuum of mass, $\mathcal{D}$, of risk-averse "depositors"; and a continuum of mass $\mathcal{E}$ of risk-neutral "equity holders." Each investor—depositor or equity holder—is endowed with one unit of capital, while banks and entrepreneurs have no capital. All agents are protected by limited liability. We use lowercase letters to denote choices by individual agents—depositors, equity holders and entrepreneurs—and uppercase letters to denote aggregate variables.

Banks raise capital from depositors and equityholders, and invest their capital in firms. Firms raise capital from banks and equityholders. There is a mass, $\mathcal{S}$, of ex ante identical "sectors" in the economy with depositors, banks and firms being distributed equally among the sectors. Hence, each sector has a mass$1/\mathcal{S}$ of banks and $\mathcal{F}/\mathcal{S}$ of firms. Banks and firms specialize within a sector: each bank lends to entrepreneurs within its sector, and each entrepreneur obtains capital from a bank within its sector. As our subsequent model development makes clear, sectoral segmentation is a convenient and tractable way to distinguish between sectoral and aggregate shocks as well as incorporate the imperfect diversification of banks. We avoid taking a specific position on the formation and composition of sectors, which can arise from various sources such as geographical proximity and industry specialization, because it is not important to our analysis. We assume the sectors are ex ante identical purely to simplify the notation. We can allow for heterogeneous sectors without altering our results.

Figure 1 shows the decisions of agents in the model. Equity holders, depositors and banks have access to a liquid "safe asset" that is in perfectly elastic supply (or, alternatively, a linear storage technology) and provides a return (per unit of capital invested) R_f, that we normalize to 1. Each depositor invests her capital in a portfolio comprising of the safe asset and deposits of a single bank. Equity holders invest their capital in a portfolio comprising of the safe asset as well as equity stakes in banks and firms.

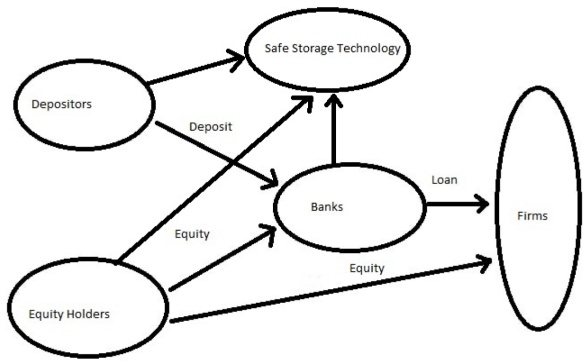

Figure 1: The graph illlustrates the investment opportunities of Depositors, Equity Holders and Banks.

Banks raise capital through equity and deposits and invest their capital in a portfolio comprising of firms and the safe asset. Each entrepreneur is endowed with a project with decreasing returns to scale and raises capital for her project from banks and equity holders. Capital markets—deposit and equity markets—are competitive, that is, depositors, equity holders, banks and firms take the returns on equity and deposits as given in making their capital demand and supply decisions. Bank loan markets are also competitive; banks and firms take the loan interest rate as given in making their supply and demand decisions, respectively.

3.1 Entrepreneurs/Firms

Each risk-neutral entrepreneur/firm is endowed with a single risky project that has decreasing returns to scale. An investment of k units of capital at date 1 yields a payoff of $\Lambda(k)$ if the project "succeeds" with probability $q \in (0,1)$, or 0 if the project "fails" with probability $1-q$ at date 2, where $\Lambda(.)$ is strictly increasing, concave and twice continuously differentiable. The projects of different firms are identically distributed (that is, they have the same success probability q), but are *not independent* as we discuss shortly. We assume that $\Lambda(0) = 0$ and $\Lambda(.)$ satisfies the Inada conditions, $\Lambda'(0) = \infty, \Lambda'(\infty) = 0$, which ensure the existence of an interior solution to an entrepreneur's capital demand.

Entrepreneurs can obtain capital from banks and equity holders so the real sector of the economy obtains financing from banks, but also competes with banks for equity capital. The bank loan market is determined by a *single* loan rate , R_L, that determines a feasible contract between a bank and a firm.[2] Banks and entrepreneurs take the loan rate, R_L, as given in making their capital supply and demand decisions, respectively. Given the payoff structure of a firm's project, a loan contract is "isomorphic" to an equity contract in that both contracts only pay off upon the project's success.[3] Further, in a nontrivial interior equilibrium, equity holders are indifferent between supplying their capital to banks and firms, and firms are indifferent between obtaining financing from banks and equity holders. (We provide the simple sufficient condition that ensures an interior equilibrium in Section 4.) For simplicity, therefore, we hereafter assume that the contract between a firm and an equity holder is also defined by the rate R_L.

A firm's demand for capital, therefore, solves

$$k = \arg\max_{k'}(\Lambda(k') - R_L k') = \left(\Lambda'\right)^{-1}(R_L) \tag{1}$$

Hence, the firm's payoff if its project succeeds is

$$\text{Firm Payoff if Project Succeeds} = \Lambda(k) - k\Lambda'(k), \tag{2}$$

which is strictly positive because $\Lambda(0) = 0$, $\Lambda(.)$ is strictly concave and satisfies the Inada conditions. Hence, for a given loan rate, R_L, all firms demand the same amount of capital. The revenue of a bank or an equity holder from a single firm if its project succeeds is

$$\Gamma(k) = R_L k = \Lambda'(k)k \tag{3}$$

[2]We note here that, because banks match with entrepreneurs within the same sector, the loan rate could vary across sectors in principle. However, because the sectors are ex ante identical, the loan rates are the same across sectors in equilibrium. In fact, the equilibrium loan rates are the same across sectors even in a model with heterogeneous sectors as long as the relative proportions of depositors, banks and firms do not vary across sectors. Without loss of generality, therefore, we assume that the loan rate is the same across sectors to simplify the notation and exposition. It is worth emphasizing, however, that the broad implications of our study would also hold in a model in which the equilibrium loan rates vary across sectors.

[3]Alternatively, we can also assume that firms obtain financing from banks via "bank debt" and outside investors via "corporate debt" without altering any of our implications.

3.2 Sectoral Risk

The projects of different firms in a sector are exposed to sectoral shocks and are, therefore, not independent of each other. Specifically, with probability p, which is the same for all sectors, a proportion ω_H of all firms in a sector succeed, and with probability $1-p$, a proportion $0 \leq \omega_L < \omega_H \leq 1$ succeed. The probability p determines the distribution of the sectoral shock that differs in general from the success probability q of an individual firm. Because all firms have the same success probability q, it follows by the law of large numbers that

$$q = p\omega_H + (1-p)\omega_L. \tag{4}$$

3.3 Banks

Each bank, $m \in [0,1]$, is financed by deposits (D_m) and equity (E_m) and it chooses how much to invest in firms (L_m) and the safe asset (S_m). A bank's balance sheet must satisfy the accounting identity

$$L_m + S_m = D_m + E_m. \tag{5}$$

The bank is fully diversified within its sector and is, therefore, only exposed to *sectoral shocks*, and not firm-specific shocks of firms within the sector. Because banks are ex ante identical, the firms in a bank's loan pool inherit the correlation structure of firms in the sector. With probability p, which is the same for all banks, a proportion ω_H of the firms in the bank's loan pool succeed, and with probability $1-p$, a proportion $0 \leq \omega_L < \omega_H \leq 1$ succeed. It follows from (3) that the bank's total revenue from its loan pool is $\omega_H L_m R_L$ with probability p, and $\omega_L L_m R_L$ with probability $1-p$. For expositional convenience, we hereafter refer to the two possible states of the bank's loan pool as "success" and "failure", respectively. Therefore, the bank's return on its loan pool is $\tilde{R}_L = (\omega_H R_L, \omega_L R_L)$.

3.4 Aggregate Risk

The economy is exposed to aggregate risk so the shocks to different sectors are not necessarily independent. Specifically, a proportion τ of sectors is exposed to an aggregate shock so that all these sectors *simultaneously* experience positive or negative shocks. The shocks faced by the remaining proportion $1-\tau$ of sectors are independent of each other. Since an individual sector faces a binomial shock with success probability p, it follows that, with probability p, a proportion ω_H of all firms in these sectors simultaneously succeed and, with probability $1-p$, a proportion ω_L of all firms in these sectors simultaneously succeed. Hence, by the law of large numbers, following a positive realization of the aggregate shock, a proportion γ_H of all firms in the economy succeed, and a proportion $1-\gamma_H$ fail. With probability $1-p$, a proportion γ_L of all firms in the economy succeed, and a proportion $1-\gamma_L$ fail. Here,

$$\gamma_H = [\tau + p(1-\tau)]\,\omega_H + [(1-p)(1-\tau)]\,\omega_L; \tag{6}$$

$$\gamma_L = [p(1-\tau)]\,\omega_H + [\tau + (1-p)(1-\tau)]\,\omega_L; \tag{7}$$

All agents in the economy observe τ, but agents (including banks) do not know a priori which sectors are exposed to the aggregate shock. An individual bank makes its financing and loan decisions based on its probability of success, p. As we discuss later, the regulator, however, internalizes the aggregate risk of the economy that is represented by the proportion τ of sectors who are exposed to the aggregate shock.

3.5 Investors

Equity holders are risk-neutral and invest their capital in a portfolio of the safe asset, bank equity and firm equity. The bank equity market is characterized by a set $\left\{\tilde{R}_E^m\right\}; m \in [0,1]$ of equity contracts, where $\tilde{R}_E^m$ is the return on the equity contract offered by bank m. $\tilde{R}_E^m$ is, in general, a random variable whose realization depends on the return on the bank's assets. Equity markets are competitive so equity holders and banks take the equity returns, $\left\{\tilde{R}_E^m\right\}; m \in [0,1]$, offered by all banks as given in making their equity capital supply and demand decisions, respectively. As noted earlier, the firm equity market is determined by the rate, R_L, which is the return on firm equity if the firm succeeds with the equity return being zero if the firm fails. Because equity holders are risk-neutral, an equity holder invests all his capital in the asset—the safe asset, bank equity or firm equity—that provides the highest expected return. If multiple banks and/or firms offer the same (maximum) expected equity return, then equity holders are indifferent between them, and invest in any number of banks and/or firms offering the same maximum expected equity return to ensure that capital markets clear.

Depositors have a strictly increasing, strictly concave and twice differentiable utility function $u(.)$. Each depositor invests her capital in a portfolio comprising of the safe asset and bank deposits. The deposit market is competitive and is characterized by a *single* deposit rate, R_D, which is the return (per unit of capital invested) from bank deposits *provided* the bank is able to meet its deposit liabilities. Banks are protected by limited liability. If a bank is unable to fully meet its liabilities, its assets are allocated among its depositors in a pro rated manner. To accommodate the possibility of bank default, we formally characterize the deposit market by a set $\left\{\tilde{R}_D^m\right\}; m \in [0,1]$ of deposit contracts, where $\tilde{R}_D^m$ is the return on the deposit contract offered by bank m. Here, $\tilde{R}_D^m$ is a random variable that equals the deposit rate, R_D, if bank m does not default on its liabilities, and is strictly less than R_D if the bank defaults. Analogous to the equity market, depositors and banks take the deposit returns, $\left\{\tilde{R}_D^m\right\}; m \in [0,1]$, offered by all banks as given in making their deposit capital supply and demand decisions, respectively. Each depositor invests a portion of her capital in the safe asset and the remaining portion in deposits of a *single* bank that provides her with the maximum expected utility. If multiple banks offer the same (maximum) expected utility to the depositor, she randomly selects a single bank to invest in with her choice being drawn from the uniform distribution over the set of banks that offer the maximum expected utility. We implicitly assume here that there are transaction costs (e.g., search costs, differences in geographic proximity, etc) that prevent a depositor from allocating her capital across multiple banks.

Each depositor chooses the proportions β and $1-\beta$ of her capital to invest in bank deposits and the safe asset, respectively, to maximize his expected utility, that is, each depositor's allocation choice satisfies

$$d(\widetilde{R}_D^m) = \arg\max_{0\leq\beta\leq 1} E\left[u((1-\beta)+\beta\widetilde{R}_D^m)\right]. \tag{8}$$

Since u is continuously differentiable, we can see that the supply function d is continuous.

3.6 Realized Returns on Deposits and Equity

Consider a bank $m \in [0,1]$ with deposit capital D_m, equity capital E_m, loan capital L_m, and safe asset holdings S_m. The variables must satisfy the constraint (5). The end-of-period gross payoff (before payments to depositors) is

$$\text{Gross Payoff} = \tilde{\rho}_m \begin{cases} = & \omega_H L_m R_L + S_m \text{if loan pool succeeds} \\ = & \omega_L L_m R_L + S_m \text{ if loan pool fails.} \end{cases} \tag{9}$$

If the deposit rate is R_D (the return on deposits unless the bank defaults), the total realized payoff of all the depositors of the bank m is as follows.

$$\text{Depositors' Payoff} = \tilde{\xi}_m \begin{cases} = & min(\omega_H L_m R_L + S_m, D_m R_D) \text{ if the bank's loan pool succeeds} \\ = & min(\omega_L L_m R_L + S_m, D_m R_D) \text{ if the bank's loan pool fails} \end{cases} \tag{10}$$

Since the bank's equity holders are residual claimants, their total payoff is

$$\text{Equityholders' Payoff} = \tilde{\rho}_m - \tilde{\xi}_m. \tag{11}$$

The realized returns on deposits and equity are, therefore, given by

$$\tilde{R}_D^m = \frac{\tilde{\xi}_m}{D_m}; \tilde{R}_E^m = \frac{\tilde{\rho}_m - \tilde{\xi}_m}{E_m}. \tag{12}$$

Note that the realized return, , $\tilde{R}_D^m$ on deposits is, in general, a random variable that differs from the deposit rate, R_D, if the bank is unable to fully meet its deposit liabilities. If the bank is able to fully pay off depositors in all states, then $\tilde{R}_D^m \equiv R_D$.

3.7 Equilibrium Conditions

An equilibrium of the unregulated economy is characterized by (i) a bank deposit rate, R_D^*; (ii) a set, $\left\{\tilde{R}_D^{m*}\right\}; m \in [0,1]$ of deposit contracts where $\tilde{R}_D^{m*} \leq R_D^*$ and $\tilde{R}_D^{m*} < R_D^*$ iff the bank m defaults; (iii) a set, $\left\{\tilde{R}_E^{m*}\right\}; m \in [0,1]$ of equity contracts; and (iv) a loan rate, R_L^* such that the following conditions are satisfied:

1. *Depositor Decisions:* Each depositor chooses her allocation of capital to the safe asset and bank deposits to maximize her expected utility taking the set, $\left\{\tilde{R}_D^{m*}\right\}; m \in [0,1]$, of deposit contracts as given. The *promised* deposit returns, $\left\{\tilde{R}_D^{m*}\right\}; m \in [0,1]$, coincide with the *realized* deposit returns defined by (10) and (12).

2. *Equity holder Decisions:* Each equity holder chooses her portfolio of investments in the safe asset, banks and firms taking the returns of the set of bank equity contracts, $\left\{\tilde{R}_E^{m*}\right\}; m \in [0,1]$, and the loan rate, R_L^*, as given. The *promised* bank equity returns, $\left\{\tilde{R}_E^{m*}\right\}; m \in [0,1]$, coincide with

the *realized* equity returns defined by (11) and (12).

3. *Bank Decisions:* Each bank chooses how much capital to raise via deposits and equity, and how much capital to allocate to the safe asset and its loan portfolio to maximize its expected profit taking the set of deposit returns, equity returns, and the loan interest rate as given.

4. *Entrepreneur Decisions:* Each entrepreneur chooses how much capital to borrow from banks and/or firms taking the loan interest rate, R_L^*, as given.

5. *Limited Liability:* All agents are protected by limited liability so their payoffs must be non-negative.

6. *Market Clearing:* Equity markets, bank deposit markets and loan markets clear.

4 Unregulated Equilibria

As we show below, all equilibria are symmetric where banks raise the same amount of equity and deposit capital, and make identical investment decisions. Depending on parameter values, unregulated equilibria can feature either default by banks when their loan portfolios fail, or no default because the values of banks' assets are sufficient to pay off depositors in full. We restrict consideration to the parameter constellations for which unregulated equilibria feature bank default. (We derive the conditions for such equilibria in the following.) We can show that equilibria where banks do not default are efficient so there is no need for regulatory intervention.

We make the following standing assumption.

Assumption 1

$$q\Lambda'(\frac{\mathcal{E}}{\mathcal{F}}) > 1,\ q\Lambda'(\frac{\mathcal{E}+\mathcal{D}}{\mathcal{F}}) < 1. \tag{13}$$

The first inequality in (13) states that, if all the equity capital, $\mathcal{E}$, in the economy is invested in firms, but no deposit capital is invested so each firm obtains capital, $\frac{\mathcal{E}}{\mathcal{F}}$ (as the mass of firms in the economy is $\mathcal{F}$, the expected marginal return from production, $q\Lambda'(\frac{\mathcal{E}}{\mathcal{F}})$, exceeds one. The second inequality states that, if all the available capital in the economy—deposit and equity capital, $\mathcal{E}+\mathcal{D}$—is invested in firms, then the expected marginal return from production, $q\Lambda'(\frac{\mathcal{E}+\mathcal{D}}{\mathcal{F}})$, is less than 1. The two conditions together ensure the existence of "interior" equilibria where the representative bank raises deposit and equity financing; equity holders are indifferent between investing in banks and firms; and firms are indifferent between obtaining capital from banks and equity holders.

The following lemma provides a simple "no arbitrage" condition on the expected returns of bank securities and loans.

Lemma 1 *In any unregulated equilibrium where banks default with a positive probability, the expected returns on bank equity, firm equity, bank deposits, and bank loans are equal and greater than the return on the safe asset (that we normalized to 1). Moreover, banks do not invest in the safe asset.*

In an interior equilibrium, the expected returns of all securities must be equal to avoid arbitrage. Further, Assumption 1 ensures that the expected loan return exceeds one (the safe asset return) so banks do not invest in the safe asset in equilibrium. The above lemma immediately implies that all unregulated equilibria are symmetric where all banks have the same expected equity, deposit and loan returns. Further, because the expected deposit and equity returns are identical across banks, banks have the same capital structures, that is, they raise the same amounts of equity and deposit capital.

We now specify the set of conditions that determine an unregulated equilibrium. We use the superscript '$unreg$' to denote equilibrium variables in the unregulated economy. When the representative bank's portfolio fails, the value of its assets is insufficient to pay off depositors in full so bank deposits are risky. Let $\widetilde{R}_D^{unreg} \equiv (R_D^{s,unreg}, R_D^{f,unreg})$ denote the equilibrium random return on the representative bank's deposits, where $R_D^{s,unreg}$ is the return when the representative bank's assets succeed and $R_D^{f,unreg}$ is the return when the assets fail and the bank is unable to pay off depositors in full. Similarly, let $\widetilde{R}_E^{unreg} \equiv (R_E^{s,unreg}, R_E^{f,unreg})$ be the equilibrium random return on the representative bank's equity.

Theorem 1 (Default Equilibria) *Default equilibria are determined by the following conditions:*

$$\Lambda'(\frac{X^{\text{unreg}}}{\mathcal{F}}) = R_L^{\text{unreg}};\ X^{\text{unreg}} = \mathcal{E} + D^{\text{unreg}} \quad (14)$$

$$L^{\text{unreg}} = E^{\text{unreg}} + D^{\text{unreg}};\ S^{\text{unreg}} = 0 \quad (15)$$

$$E[\tilde{R}_E^{\text{unreg}}] = E[\tilde{R}_D^{\text{unreg}}] = qR_L^{\text{unreg}} > 1 \quad (16)$$

$$D^{\text{unreg}} = \mathcal{D}d(\tilde{R}_D^{\text{unreg}}) = \mathcal{D}d(R_D^{\text{s,unreg}}, R_D^{f,\text{unreg}}) \quad (17)$$

$$R_D^{f,\text{unreg}} D^{\text{unreg}} = \omega_L R_L^{\text{unreg}} L^{\text{unreg}};\ R_D^{f,\text{unreg}} < R_D^{s,\text{unreg}} \quad (18)$$

$$R_E^{s,\text{unreg}} = \frac{\omega_H R_L^{\text{unreg}} L^{\text{unreg}} - R_D^{\text{unreg}} D^{\text{unreg}}}{E^{\text{unreg}}};\ R_E^{f,\text{unreg}} = 0 \quad (19)$$

In equilibrium, the total capital, $\mathcal{E}$, held by equity holders is supplied to banks and firms. If the total capital supplied by depositors to banks is D^{unreg}, it follows from the fact that banks invest all their capital in firms that the total capital invested in firms either by banks or by equity holders is $X^{unreg} = \mathcal{E} + D^{unreg}$. Because each firm demands the same amount of capital for a given loan rate by (1), the total capital invested in firms is equally allocated among the mass $\mathcal{F}$ of firms so that each firm obtains capital $\frac{X^{unreg}}{\mathcal{F}}$. It then follows from (1) that the equilibrium loan rate, R_L^{unreg}, is given by the first equation in (14). Conditions (15) represent the representative bank's budget balance condition, and the fact that the representative bank does not invest in the safe asset in equilibrium. Equation (16) states that the expected returns on bank deposits, equity and loans are equal in equilibrium. The equalities, (17), imply that the total deposit capital supplied to banks, D^{unreg}, must be given by the total capital, $\mathcal{D}$, held by depositors multiplied by the optimal supply of capital by the representative depositor to a bank given the deposit return, $\widetilde{R}_D^{unreg}$, that is given by (8). Conditions (18) and (19) express the fact that the assets of defaulted banks are distributed to their depositors, and that equity holders are residual claimants to a bank's payoff upon success (that is, after payments to depositors).

The following proposition establishes that there is a continuum of unregulated equilibria, where each equilibrium is determined by the total capital that depositors supply to banks.

Proposition 1 (Set of Default Equilibria) *There exists an interval $[D_{min}, D_{max}) \subset (0, \mathcal{D})$, such that an equilibrium with risky deposits exists for all $D^{\text{unreg}} \in [D_{min}, D_{max})$, where D^{unreg} is the total capital supplied by depositors to banks. The size of the representative bank, and the total production in the economy, increases with D^{unreg}, while the expected return on bank deposits, equity and loans decreases* *with* D^{unreg}*. Moreover, when* $D^{\text{unreg}} = D_{min}$*,* $E^{\text{unreg}} = 0$*, and when* $D^{\text{unreg}} = D_{max}$*,* $E^{\text{unreg}} = \mathcal{E}$*.*

Equation (14) in Theorem 1 shows that an unregulated equilibrium is associated with a loan rate, R_L^{unreg}, that is itself determined by the total capital, X^{unreg}, invested in firms, As we discussed above, the total equity capital, $\mathcal{E}$, in the economy is invested in firms either directly by equity holders or indirectly via banks. Consequently, the total capital invested in firms is determined by the total capital, D^{unreg}, that depositors supply to banks. Hence, each unregulated equilibrium is determined by the total capital that depositors supply to banks.As the expected deposit return exceeds one, depositors supply nonzero capital to banks so there can be no equilibrium where depositors do not invest in banks, that is, $D_{min} > 0$. By Assumption 1, there is also *no equilibrium* in which depositors supply *all their capital* to banks, which corresponds to $D^{unreg} = \mathcal{D}$. Indeed, in this scenario, if the capital, $\mathcal{E} + \mathcal{D}$, were invested in firms, it follows from (13) and (14) that the expected loan return would be less than one so that it would be optimal for banks to invest some capital in the safe asset, which contradicts Lemma 1.

If D^{unreg} is the total deposit capital in any candidate equilibrium, then (14) uniquely determines the expected loan return, which equals the expected return on bank deposits by Lemma 1. The equilibrium condition (17) then uniquely determines the random deposit return, $\widetilde{R}_D^{unreg} \equiv (R_D^{s,unreg}, R_D^{f,unreg})$, so the total capital raised by banks, L^{unreg}, is determined by (18). The candidate equilibrium is, indeed, an equilibrium iff the total capital raised by banks is (i) greater than or equal to the total deposit capital supplied to banks, but (ii) less than the total deposit capital supplied to banks plus the total available equity capital. In other words,

$$D^{unreg} + \mathcal{E} \geq L^{unreg} \geq D^{unreg}. \tag{20}$$

The above inequalities determine an interval of possible values of the total deposit capital supplied to banks. Hence, there is an equilibrium corresponding to each possible value of the total deposit capital in this interval. As the total deposit capital in equilibrium, D^{unreg}, increases, the total capital invested in firms, $X^{unreg} = \mathcal{E} + D^{unreg}$, increases. This, in turn, lowers the expected loan return by (14) and, therefore, the expected return on bank deposits. In other words, an increase in deposit capital is associated with a *lower risk premium* on bank deposits so that deposits become *less risky*. For banks to receive greater deposit capital that is also less risky, however, they must also raise more equity capital to serve as a buffer to pay back depositors when their assets yield low returns. Hence, an increase in the equilibrium deposit capital is associated with an increase in equity capital and, therefore, a larger bank size. The equilibrium corresponding to the maximum possible deposit capital, D_{max}, is the one for which the economy invests the maximum amount of capital in production. As we show in the proof of the proposition, banks raise all available equity capital in the economy in this equilibrium. That is, this equilibrium corresponds to a "full intermediation" equilibrium in which firms (the real economy) are financed entirely by banks. Consequently, the unregulated equilibrium with the *smallest* banks features the *riskiest* deposits, while the equilibrium with the *largest* banks features the *safest* deposits.This suggests that regulatory policy that primarily serves to constrain bank size should be viewed with caution.

Depositor *risk aversion* and the *strict concavity* of firms' production technologies play key roles in driving the properties of unregulated equilibria. The benefit of bank debt from the standpoint of depositors arises due to the *endogenous premium* that banks are able to provide risk-averse depositors by investing their capital in value-enhancing production. The cost of bank debt stems from the utility loss to risk-averse depositors due to bank insolvency. The debt cost-benefit tradeoff leads to a range of unregulated equilibria with nontrivial bank capital structures even in the absence of traditional frictions that lead to nonzero leverage. A concave production technology implies that bank size is pinned down in each unregulated equilibrium, and unregulated equilibria differ in the sizes of the financial and real sectors of the economy.

Proposition 1 *does not imply* that banks' capital structure is irrelevant as in the Modigliani-Miller theorem, but only that there are multiple unregulated equilibria that feature different bank sizes and capital structures. The vartiation in bank size across the different equilibria stems from the link between banks' financing and investment decisions. As we show in Section 6, regulatory intervention breaks the equilibrium multiplicity. More precisely, for a given set of values of regulatory parameters, the equilibrium of the regulated economy is *unique*.

5 Efficient Allocations

There are three potential inefficiencies in the unregulated economy. First, consistent with the evidence in Guiso et al. (2002), there is limited market participation as depositors cannot directly invest in productive firms, but only indirectly via banks. Second, each depositor invests in a single bank and is, therefore, exposed to bank-specific risk. Further, banks' portfolios are not fully diversified. Third, agents do not internalize the effects of the aggregate shock when they make their investment decisions. The above inefficiencies lead to imperfect diversification of depositors and banks, and incomplete sharing of aggregate risk among depositors, equity holders and entrepreneurs. Further, as we show below, limited market participation by depositors as well as the incomplete internalization of aggregate risk can lead to under- or over-investment of capital in production depending on the level of aggregate risk in the economy.

We now derive the efficient allocations in the economy by considering the problem of a *hypothetical* social planner. The social planner invests households' capital in entrepreneurs/firms and the safe asset. The planner then distributes the payoffs from investments among depositors, equity holders and entrepreneurs. Banks are risk-neutral financial intermediaries and, therefore, receive no payoff in an efficient allocation. Given that depositors are risk-averse, while all other agents are risk-neutral, we assume that the social planner's objective is to maximize the depositors' expected utility subject to keeping the payoffs of the other agents—the equity holders and the entrepreneurs—no less than the their payoffs in the unregulated equilibrium. As we showed in the previous section, there is a continuum of unregulated equilibria determined by an interval of possible values of the deposit capital raised by the representative bank. We derive the *set* of efficient allocations corresponding to the *set* of unregulated equilibria. In other words, for *each* unregulated equilibrium, we characterize the efficient allocation in which the expected utility of depositors is maximized subject to the payoffs of equity holders and entrepreneurs being no less than their respective payoffs in the unregulated equilibrium.

Accordingly, consider any unregulated equilibrium in which the representative bank has equity, debt and total capital, $(e^{unreg}, d^{unreg}, l^{unreg})$, and the expected return on bank deposits, equity and loans is $E[\widetilde{R}^{unreg}]$. The expected payoffs of entrepreneurs and equity holders in this unregulated equilibrium, denoted by Δ_F and Δ_E, respectively, are:

$$\Delta_E = E[\widetilde{R}^{unreg}]\mathcal{E} \tag{21}$$

$$\Delta_F = q\mathcal{F}\Lambda(\frac{X^{unreg}}{\mathcal{F}}) - E[\widetilde{R}^{unreg}]X^{unreg} \tag{22}$$

Because it is inefficient for agents to be exposed to firm- and sector-specific risks, the efficient allocation only depends on the aggregate state. Let us denote the set of aggregate states of the economy by $\Omega \equiv \{H, L\}$. As described in Section 3.4, the state H is the "positive" aggregate state where a proportion γ_H of all firms succeed, and L refers to the "negative" aggregate state where a proportion γ_L of all firms succeed, where γ_H, γ_L are defined in (6) and (7), respectively. Suppose that the social planner invests X_S and X_F in the safe asset and entrepreneurs' projects, and distributes P_D^ω to the deposit holders, P_E^ω to the equity holders and P_F^ω entrepreneurs in the state $\omega \in \Omega$. The planner's objective is then

$$\max_{X_F, X_S, P_D^L, P_D^H, P_E^L, P_E^H \geq 0} E[u(P_D^\omega)] \text{ subject to}$$

$$X_S + X_F \leq \mathcal{D} + \mathcal{E} \tag{23}$$

$$P_D^H + P_E^H + P_F^H \leq \gamma_H \mathcal{F} \Lambda(\frac{X_F}{\mathcal{F}}) + X_S \tag{24}$$

$$P_D^L + P_E^L + P_F^L \leq \gamma_L \mathcal{F} \Lambda(\frac{X_F}{\mathcal{F}}) + X_S, \tag{25}$$

$$E[P_E^\omega] \geq \Delta_E; E[P_F^\omega] \geq \Delta_F \tag{26}$$

Equation (23) is the aggregate resource constraint. Equations (25) and (24) mean that the total capital allocated to all agents does not exceed the total available capital in the low and high states, respectively. Because a proportion γ_H of firms succeeds in the high state, the total revenue of all firms in the high state is $\gamma_H \mathcal{F} \Lambda(\frac{X_F}{\mathcal{F}})$. Similarly, the total revenue of all firms in the low state is $\gamma_L \mathcal{F} \Lambda(\frac{X_F}{\mathcal{F}})$. The constraints, (26) specify that the total allocated capital to equity holders and entrepreneurs is no less than their payoffs in the unregulated equilibrium.

We note that an unregulated equilibrium that features safe deposits is efficient because depositors bear no risk so their utility trivially attains its maximum possible level, while ensuring that equityholders and entrepreneurs together receive at least their expected payoffs in the unregulated equilibrium. We, therefore, examine the more interesting scenario where the unregulated equilibria feature risky deposits. We use the superscript, "eff", to denote variables in the efficient allocation.

Theorem 2 (Efficient Allocations with Risky Deposits) *There exists a threshold, $\bar{\tau}$, of the aggregate risk that determines the efficient allocation, where*

$$\bar{\tau} = \frac{\Delta_E + \Delta_F}{p \mathcal{F} \Lambda(\frac{X_F^{eff}}{\mathcal{F}})(\omega_H - \omega_L)}. \tag{27}$$

In the above, X_F^{eff}, is the efficient level of investment in firms.

1. Full Insurance: *If $\tau \leq \bar{\tau}$, then the depositors receive the same allocation in the low and high states, that is, they receive full insurance against aggregate risk so that $P_D^{L,eff} = P_D^{H,eff}$. The total capital, X_F^{eff}, invested in firms is determined by*

$$\Lambda'(\frac{X_F^{eff}}{\mathcal{F}}) = \frac{1}{q}. \tag{28}$$

The expected marginal return , on investment in firms, $E[\widetilde{R}^{eff}]$, equals the safe asset return.

$$E[\widetilde{R}^{eff}] = [p\gamma_H + (1-p)\gamma_L]\,\Lambda'(\frac{X_F^{eff}}{\mathcal{F}})) = q\Lambda'(\frac{X_F^{eff}}{\mathcal{F}})) = 1. \tag{29}$$

$$X_S^{eff} = \mathcal{E} + \mathcal{D} - X_F^{eff} \tag{30}$$

The expected return to depositors, $E[\widetilde{R_D}^{eff}]$, is

$$E[\widetilde{R_D}^{eff}] = \frac{P_D^{L,eff}}{\mathcal{D}} = \frac{P_D^{H,eff}}{\mathcal{D}} > 1 \tag{31}$$

2. Incomplete Insurance: *If $\tau > \bar{\tau}$, then depositors bear aggregate risk so that $P_D^{L,eff} < P_D^{H,eff}$. Depositors receive the entire available output in the economy in the low aggregate state, while equityholders and entrepreneurs get nothing. The total investment, X_F^{eff}, in firms is determined by the following first order condition:*

$$\begin{aligned}&\frac{\partial}{\partial X_F}[pu((\gamma_H \mathcal{F}\Lambda(\frac{X_F}{\mathcal{F}}) + \mathcal{D} + \mathcal{E} - X_F - \frac{\Delta_E + \Delta_F}{p})\\ &+(1-p)u(\gamma_L \mathcal{F}\Lambda(\frac{X_F}{\mathcal{F}}) + \mathcal{D} + \mathcal{E} - X_F)] = 0\end{aligned} \tag{32}$$

The expected marginal return on investment in firms is greater than the safe asset return.

$$E[\widetilde{R}^{eff}] = [p\gamma_H + (1-p)\gamma_L]\,\Lambda'(\frac{X_F^{eff}}{\mathcal{F}})) > 1. \tag{33}$$

The expected return to depositors, $E[\widetilde{R_D}^{eff}]$, is given by

$$E[\widetilde{R_D}^{eff}] = \frac{pP_D^{H,eff} + (1-p)P_D^{L,eff}}{\mathcal{D}} > 1 \tag{34}$$

The social planner faces a tradeoff between providing full insurance to risk-averse depositors against aggregate shocks, and investing in production by firms. If aggregate risk is below a threshold, the wedge between the total output in the high and low aggregate states is low enough that the social planner can fully insure depositors, while ensuring that the expected payoffs to equity holders and entrepreneurs equal their respective expected payoffs in the unregulated equilibrium. In this scenario, the social planner chooses the investment level to maximize the expected payoff from production net of investment. The planner invests the remaining capital in the safe asset to ensure that depositors are fully insured. By (29), we see that, in the efficient allocation, the amount of capital invested in firms is such that expected marginal return on investment in firms and the safe asset return are equalized. However, by (31), the expected return to deposits exceeds the safe asset return. As we see in Section 6), this implies that, in the implementation of the efficient allocation via regulation, depositors invest all their capital in banks, and receive subsidies via deposit insurance and tax transfers to ensure that their expected return exceeds one.

If aggregate risk is above a threshold, the difference between the total outputs in the low and high states is large. Consequently, it is socially costly to fully insure depositors so they must bear some aggregate risk. As equity holders and entrepreneurs are risk neutral, it is optimal for them to bear the maximum possible risk in the efficient allocation. Hence, depositors receive *all* available capital in the low aggregate state, while entrepreneurs and equity holders receive nothing. This allocation of capital minimizes the risk faced by depositors, which in turn maximizes their expected utility. In this situation, total production is below the level in the full insurance scenario because investing too much capital in production imposes excessive risk on depositors.

As the above discussion clarifies, the presence of aggregate risk and a *strictly concave* production technology generates a nontrivial tradeoff between providing insurance to risk-averse depositors that protects them against aggregate shocks, and investing in production. In contrast, in models with risk-neutral agents and linear technologies, the social objective is to maximize expected production. We now show how the efficient level of investment in entrepreneurial firms, X_F^{eff}, is affected by the aggregate risk, τ .

Proposition 2 (Aggregate Risk and Production) *The efficient level of investment, X_F^{eff}, does not vary with τ for $\tau \leq \bar{\tau}$, but decreases with τ for $\tau > \bar{\tau}$.*

When the aggregate risk is below the threshold, $\bar{\tau}$, the social planner simply maximizes expected total production net of investment so that the investment decision is independent of the aggregate risk parameter τ. If the aggregate risk is above the threshold, an increase in the aggregate risk causes the social planner to lower investment in production and invest more capital in the safe asset to offset the effects of higher aggregate risk on depositors' expected utility. The following proposition shows that, depending on the level of aggregate risk, the unregulated economy could feature either underinvestment or overinvestment relative to the efficient allocation.

Proposition 3 (Efficiency of Investment in Unregulated Equilibrium) *There exists a threshold, $\widehat{\tau} \geq \bar{\tau}$ such that the unregulated economy features underinvestment relative to the efficient allocation if $\tau < \widehat{\tau}$ and overinvestment if $\tau > \widehat{\tau}$.*

When aggregate risk is low, risk-averse depositors invest too much capital in the safe asset in the unregulated equilibrium relative to the efficient allocation where risk is optimally shared between depositors and the risk-neutral agents in the economy (equity holders and entrepreneurs). As a result, greater capital is invested in risky production in the efficient allocation. When aggregate risk is above a threshold, risk-neutral banks in the unregulated economy collectively invest too much capital in risky production because banks do not internalize the effects of aggregate risk in creating correlation among banks' asset portfolios when they make their investment decisions. In this scenario, it is optimal for the social planner to invest greater capital in the safe asset relative to the unregulated economy, which implies that there is overinvestment in production in the unregulated economy. In other words, when aggregate risk is low, there is underinvestment because depositors invest *too much* capital in the safe asset. When aggregate risk is high, however, there is overinvestment because banks invest *too little* capital in the safe asset.

6 Regulation

We now examine whether the efficient allocations we characterized in the previous section can be implemented through appropriate regulatory policies. We focus on regulatory intervention tools that are salient to banking regulation; deposit insurance, capital requirements, liquidity requirements, and bailouts financed by taxation. To facilitate our discussion, we introduce a "regulator" who embodies the roles and actions of possibly distinct institutions that implement banking regulation.

6.1 Regulatory Tools

We begin by describing the possible tools the regulator can use that mimic real-world tools used in the regulation of banks.

Deposit Insurance

The regulator can provide insurance for depositors against banks' default risk. It collects the deposit insurance premium ex ante from each bank and pays a pre-specified amount ex post to insolvent banks to indemnify depositors. The indemnity payment to insolvent banks can depend, in general, on the aggregate state. Let π denote the premium per unit of deposits in the representative bank, and let μ^j for $j \in \{L, H\}$ denote the indemnity payment per unit of deposits to the bank if it is insolvent (fails) and the aggregate state is $j \in \{L, H\}$. The regulator invests the collected premia in the safe asset and makes payments to banks from the resulting deposit insurance fund.

The deposit insurance premium, π, and the indemity payments, μ^H, μ^L, are such that the insurance fund is able to fully meet the contractually specified indemnity payments in each aggregate state. Consequently, we must have

$$\pi = [\tau + (1-\tau)(1-p)]\mu^L = [(1-\tau)(1-p)]\mu^H, \tag{35}$$

where $[\tau + (1-\tau)(1-p)]$ and $[(1-\tau)(1-p)]$ are the proportions of failed banks in the negative and positive aggregate states, respectively. It immediately follows from the above that $\mu^L < \mu^H$ so depositors receive less protection in the low aggregate state.

Bailouts

The regulator can also provide additional subsidies to failed banks via bailouts that are financed through taxation. Given that $\mu^L < \mu^H$, such bailouts can provide additional protection for depositors in the low aggregate state. As the efficient allocations only depend on the aggregate state, we can assume, without loss of generality, that the bailout subsidies and taxes only depend on the aggregate state. Accordingly, when the aggregate state is $j \in \{H, L\}$, let ϕ^j denote the marginal subsidy (subsidy per unit of deposits) to failed banks; t_D^j denote the marginal tax on deposit payments made by a successful bank; t_E^j denote the marginal tax on equity payments made by a successful bank; and T_F^j denote the *lump sum* tax on entrepreneurs of a successful firm (or subsidy if it is negative). Equity payments by successful firms are subject to a marginal tax of $\frac{pt_E^j}{q}$, to ensure that equity holders are indifferent between investing in banks and firms in equilibrium. Investments by equity holders in the safe asset are subject to a marginal tax of pt_E^j. Even though equityholders do not invest in the safe asset in equilibrium, the marginal tax on investments in the safe asset must nevertheless be specified to support the regulatory equilibrium. Regulatory payments respect the limited liability of all agents, and the regulator maintains a balanced budget in all states. In the regulated equilibria that implement the efficient allocations, failed banks and firms make no equity payments, and entrepreneurs of failed firms get zero payoffs. Hence, no taxes are imposed on these payments. To avoid cluttering the notation, we incorporate this at the outset in our notation.

Liquidity and Capital Requirements

The regulator can require banks to hold a minimum proportion of capital in the safe and liquid asset. The liquidity requirement for a representative bank takes the following form:

$$s \geq \beta(e + d), \tag{36}$$

for some $1 \geq \beta > 0$. Since we have the accounting identify, $l + s = e + d$ by (5), the above inequality is equivalent to:

$$s \geq \frac{\beta l}{1 - \beta}. \tag{37}$$

Under this policy banks are required to invest at least $\frac{\beta}{1-\beta} > 0$ in the safe asset for every unit of investment in risky firms.

The regulator can also impose a minimum capital requirement on banks, which ensures that banks raise a minimum amount of equity capital as a proportion of the total capital that they invest in loans or their "risky capital". Specifically, the capital requirement is specified as

$$e \geq \theta l. \tag{38}$$

6.2 Regulated Economy

We now discuss the implementability of the efficient allocations characterized in Section 5 by a combination of the aforementioned policies. Recall from Section 5 that there is a set of efficient allocations corresponding to the set of possible unregulated equilibria. For generality, we remain agnostic about the specific efficient allocation that the regulator wishes to implement. Our results below hold for any choice of efficient allocation subject to an additional assumption that focuses attention on empirically relevant allocations in the context of bank regulation. Consider an efficient allocation

$$\Gamma \equiv \left\{ X_F^{eff}, X_S^{eff}, P_D^{L,eff}, P_D^{H,eff}, E[\widetilde{R}^{eff}], \Delta_E, \Delta_F \right\}, \tag{39}$$

where we define the vector of quantities above in Theorem 2. We assume that the available equity capital in the economy alone is insufficient to finance the efficient investment level, that is, $\mathcal{E} \leq X_F^{eff}$. This ensures a nontrivial role for bank regulation in the implementation of the efficient allocation by requiring that banks must raise deposits.

We now describe the structures of the equity, deposit and loan contracts in the regulated economy. The presence of taxes, deposit insurance payments and subsidies creates wedges between the marginal costs of equity and deposit financing for banks, and the corresponding returns to equity holders and depositors.

Recall from Section 5 that the depositors only bear aggregate risk (if at all) in an efficient allocation. Hence, the deposit contract in the regulated economy takes the form, $\widetilde{R}_D \equiv (R_D^L, R_D^H)$, where R_D^j is the return to depositors in aggregate state $j \in \{L, H\}$. The marginal cost of deposits to the representative bank—the payments made by the bank associated with deposit financing—incorporate the deposit insurance premium and payouts from the deposit insurance fund as well as marginal taxes on deposit payments and any bailout funds. In contrast with the net returns to depositors, the marginal costs of deposit financing depend on the bank's idiosyncratic state and the aggregate state because the bank's tax liability depends on its idiosyncratic state. Let $\tilde{C}_D \equiv \left\{ C_D^{i,j};\ i \in \{s, f\},\ j \in \{H, L\} \right\}$ be a random variable representing the marginal cost of deposits to the representative bank where $C_D^{i,j}$ is the marginal cost of deposits in idiosyncratic state $i \in \{s, f\}$ and aggregate state $j \in \{H, L\}$.

Banks pay a premium π for each unit of deposits that they raise ex ante. If the bank is successful, deposit payments are subject to a marginal tax t_D^j in aggregate state $j \in \{L, H\}$. Hence, the marginal cost of deposits to the bank, $C_D^{s,j}$, in state (s,j) is the *net return*, R_D^j , to depositors *plus* the marginal tax and the deposit insurance premium. If the bank fails, it receives a deposit insurance indemnity payment (per unit of deposits), μ^j and a bailout subsidy ϕ^j. Hence, the marginal cost of deposits to the bank, $C_D^{f,j}$, in state (f,j) is the *net return*, R_D^j, to depositors *plus* the deposit insurance premium *minus* the deposit insurance indemenity payment, μ^j, and bailout subsidy, ϕ^j. Therefore, the net returns to depositors and the marginal costs to the bank are related as follows.

$$\begin{aligned} R_D^H &= C_D^{s,H} - t_D^H - \pi = C_D^{f,H} - \pi + \mu^H + \phi^H, \\ R_D^L &= C_D^{s,L} - t_D^L - \pi = C_D^{f,L} - \pi + \mu^L + \phi^L. \end{aligned} \tag{40}$$

We note here that the marginal tax rate, $t_D^j; j \in \{H, L\}$, could be negative in which case depositors receive a subsidy from the regulator.

As we discussed earlier, the marginal tax on equity also depends only on the aggregate state. However, because equityholders are risk-neutral, an individual equityholder may be exposed to aggregate and idiosyncratic risk in the efficient allocation and, therefore, the regulatory equilibrium. Hence, in contrast with the net return to depositors, the net return to bank equityholders depends on both the aggregate state and the idiosyncratic state of the bank and is denoted by $\tilde{R}_E = (R_E^{f,L}, R_E^{f,H}, R_E^{s,L}, R_E^{s,H})$. If $\tilde{C}_E \equiv \left\{ C_E^{i,j};\ i \in \{s,f\},\ j \in \{H,L\} \right\}$ is the random variable denoting the marginal cost of equity financing to the representative bank, the following relation must hold between the net return to equity holders and the marginal cost of equity financing for the bank.

$$R_E^{i,j} = C_E^{i,j} - t_E^j; \text{ for } i \in \{s,f\};\ j \in \{L,H\}. \tag{41}$$

The marginal tax rate, $t_E^j; j \in \{H, L\}$ could be negative in which case equityholders receive a subsidy. As noted earlier, if bank equity payments are subject to a marginal tax of t_E^j, then firm equity payments are subject to a marginal tax of $\frac{pt_E^j}{q}$ to ensure that the expected returns to equityholders of firms and banks are equal in equilibrium as equityholders must be indifferent between investing in banks and firms.

6.3 Equilibria of Regulated Economy

To derive the equilibria of the regulated economy, we proceed as in Section 4. Analogous to the case of the unregulated economy, all agents—depositors, equityholders, banks and entrepreneurs—take the returns on deposits, (bank and firm) equity and loans as given when they make their decisions. However, the liquidity requirement, (36) and capital requirement, (38), appear as constraints on banks' decisions on their "asset" and "liability" sides, respectively. Capital markets must clear in equilibrium. We begin with the following useful lemma, which pins down the relative proportions of the representative bank's investments in the safe and risky assets, respectively, in any regulated equilibrium. We use the superscript $'reg'$ to denote equilibrium variables in the regulated economy.

Lemma 2 (Relationships Among Returns in Regulated Equilibrium) *In any regulated equilibrium, the marginal expected return on the representative bank's assets is equal to the total expected marginal cost of equity and deposit financing for the representative bank, which is its expected marginal cost of capital. The expected return on bank loans equals the expected marginal cost of equity financing. In other words, we must have*

$$\beta^{reg} + (1-\beta^{reg})(qR_L^{reg}) = \alpha^{reg} E[\tilde{C}_E^{reg}] + (1-\alpha^{reg})E[\tilde{C}_D^{reg}], \tag{42}$$

$$qR_L^{reg} = E[\tilde{C}_E^{reg}], \tag{43}$$

where β^{reg} is the liquidity requirement defined by (36) and $\alpha^{reg} = (1-\beta^{reg})\theta^{reg}$ is the proportion of equity capital in the bank's total capital in the regulated equilibrium. Further,

$$E[\tilde{C}_D^{reg}]d^{reg} = qR_L^{reg}d^{reg} + (1-qR_L^{reg})s^{reg}, \tag{44}$$

where d^{reg} and s^{reg} are, respectively, the representative bank's deposit capital and investment in the safe asset in the regulated equilibrium.

The intuition for the lemma is as follows. The expected return on the representative bank's assets equals the expected marginal cost of financing in equilibrium since the bank makes zero expected profit. If the liquidity requirement, (37), is not binding, then the bank voluntarily invests nonzero capital in the safe asset and is, therefore, indifferent between investing in the safe asset and firms. Hence, the expected loan return must equal the safe asset return, which is one. Therefore, the expected marginal costs of bank deposit and equity financing are equal to one as well snce banks make zero expected profits. In this case, the right-hand side and left-hand side of (42) and (43) are equal to one.

If the expected loan return is greater than one, banks do not voluntarily invest in the safe asset. In this case, the liquidity constraint, (37) is binding. The L.H.S. of (42) is the expected return on the representative bank's portfolio when it is required to invest a portion β^{reg} in the safe asset. The R.H.S. of (42) is the total expected marginal cost of deposit and equity financing for the bank, which is its expected marginal cost of capital. In equilibrium, equity holders are indifferent between investing in banks and firms, and firms are indifferent between bank and equity financing. Hence, the expected return on bank loans equals the expected marginal cost of equity financing for banks as expressed by (43). Equality (44) follows from (i) the equality of the expected marginal cost of equity financing and the expected return on bank loans (see (43)); and (ii) the fact that the expected total cost of deposit and equity financing to the representative bank must equal the expected total payoff from its assets, which comprise the safe asset and loans to firms. We note here that, because the equity constraint, (38), may be *binding* in a regulated equilibrium, the expected marginal costs of bank deposit and equity financing *may not be equal*. We show that this is, indeed, the case below.

We now proceed to the characterization of the regulated equilibria. The properties of the regulated equilibria depend on the unregulated equilibrium that determines the expected payoffs of equityholders and entrepreneurs in the efficient allocation as discussed in Section 5. By Theorem 2, the efficient allocation features full insurance for depositors when the aggregate risk, τ, is below the threshold $\bar{\tau}$ defined in (27) and incomplete insurance when aggregate risk is above the threshold. Let P_D^{eff} be the depositors' payoff in the efficient allocation. Deposit insurance and bailouts financed by taxation play substitutable roles in protecting depositors from banks' default risk. The greater the amount of deposit insurance purchased by banks, the less the need for ex post bailouts. This suggests that the same efficient allocation can be implemented by multiple regulatory equilibria that differ only in the amount of deposit insurance purchased by banks. The amount of deposit insurance that banks purchase is, in turn, fully determined by the deposit insurance premium (see (35)). As our following results show, the same efficient allocation can also be implemented by multiple equilibria that differ only in the liquidity requirement. However, given a liquidity requirement and deposit insurance policy (determined by the insurance premium), the other regulatory intervention tools—the capital requirement and bailouts—are pinned down *uniquely* in the implementation of the efficient allocation. Since the nature of the efficient allocation—full insurance or incomplete insurance for depositors—depends on the level of aggregate risk, we consider these scenarios separately below.

Full Insurance for Depositors

By Theorem 2, the efficient allocation features incomplete insurance for depositors if $\tau \leq \overline{\tau}$.

Theorem 3 (Regulated Equilibrium - Full Insurance for Depositors) *1. There exists a continuum of policy tools all of which implement the efficient allocation. For each (β^{reg}, π^{reg}) satisfying*

$$\beta^{reg}\mathcal{D} + \pi^{reg}\mathcal{D} \leq X_S^{eff},\ \beta^{reg}(\mathcal{D}+\mathcal{E}) + \pi^{reg}\mathcal{D} \geq X_S^{eff},\ \beta^{reg} \in [0,1], \tag{45}$$

there exist a unique corresponding capital requirement, θ^{reg}, and tax-financed bailout policy defined by the marginal tax rates on deposit payments by successful banks, $(t_D^{j,reg}); j \in \{H, L\}$; the marginal subsidies (subsidies per unit of deposits) to failed banks, $(\phi^{j,reg}); j \in \{H, L\}$; the expected marginal taxes on bank equity payments, $E[\widetilde{t}_E^{reg}]$; the expected marginal taxes on firm equity payments, $E\left[\frac{p\widetilde{t}_E^{reg}}{q}\right]$; and the expected lump sum taxes on entrepreneurs, $E[\widetilde{T}_F^{reg}]$ $j \in \{H, L\}$, that implement the efficient allocation. For a given set of policy tools, the equilibrium of the regulated economy is unique.

2. *The returns to bank depositors and equity holders, the expected marginal costs of deposit and equity financing for banks, and the loan rate in the regulated equilibrium are as follows.*

$$R_D^{H,reg} = R_D^{L,reg} = E\left[\widetilde{R}_D^{reg}\right] = \frac{P_D^{eff}}{\mathcal{D}} > 1, \tag{46}$$

$$R_E^{f,j,reg} = 0 \ \text{ for } j \in \{L, H\},\ E[\tilde{R}_E^{reg}] = \frac{\Delta_E}{\mathcal{E}} > 1, \tag{47}$$

$$E[\tilde{C}_D^{reg}] = E[\tilde{C}_E^{reg}] = qR_L^{reg} = E[\widetilde{R}^{eff}] = 1, \tag{48}$$

Banks are indifferent between investing in firms and the safe asset in equilibrium.

3. *The total amounts of equity capital, deposit capital, loan capital and investments in the safe asset*

in the regulated equilibrium are:

$$D^{reg} = D; \; E^{reg} = \frac{X_S^{eff} - \pi^{reg} D^{reg} - \beta^{reg} D^{reg}}{\beta^{reg}}, \tag{49}$$

$$L^{reg} = (1 - \beta^{reg})(E^{reg} + D^{reg}), \; S^{reg} = X_S^{eff} - \pi^{reg} \mathcal{D}. \tag{50}$$

Theorem 4 shows that there is a continuum of regulatory policies that implement the efficient allocation, but the equilibrium of the economy is unique for a *given* regulatory policy characterized by the liquidity requirement, deposit insurance premium, capital requirement and bailout policy. We explicitly characterize the regulatory policy and provide the expressions for the policy tools in terms of the fundamental parameters of the economy in the proof of the theorem.

The intuition for condition (45) is as follows. As the regulated equilibria implement the efficient allocation, it follows from (31) that depositors are fully insured, and the expected return on deposits is greater than one. Hence, depositors invest all their capital in banks. By (36), therefore, the portion, $\beta^{reg}\mathcal{D}$, of the total deposit capital must be invested in the safe asset. In addition, the total capital raised via deposit insurance premia, $\pi^{reg}\mathcal{D}$, in the deposit insurance fund is also invested in the safe asset. Because the proportion, β^{reg}, of total bank equity capital must also be invested in the safe asset by (36), the first inequality in (45) expresses the fact that feasible values of (β^{reg}, π^{reg}) must be such that the total capital raised via deposit financing and deposit insurance premia that is invested in the safe asset must be no greater than the efficient level, X_S^{eff}. Correspondingly, the second inequality in (45) implies that, if banks raise all available equity capital in the economy, then the total investment in the safe asset must be no less than the efficient level.

Because the expected return on bank loans equals the safe asset return, banks are indifferent between investing in firms and the safe asset. Hence, even without capital and liquidity requirements, there exists an equilibrium of the regulated economy in which the total amounts of capital allocated to firms and the safe asset are efficient. However, in the absence of capital and liquidity requirements, there may be other equilibria of the economy that are inefficient. Capital and liquidity requirements ensure that the equilibrium of the regulated economy is unique so that the regulatory policy *uniquely* implements the efficient allocation.

As noted above, the expected return to depositors is greater than one as shown by (46). The efficient allocation maximizes the expected utility of depositors while ensuring that equityholders and entrepreneurs receive their unregulated equilibrium payoffs. Because the expected return of equityholders is greater than one in the unregulated equilibrium by Lemma 1, it must also be greater than one in the regulated equilibria as shown by (47) (since the regulated equilibria implement the efficient allocation). By Theorem 2, if the aggregate risk, $\tau \leq \overline{\tau}$, such that the efficient allocation fully insures depositors, then the expected marginal return on investment in firms, $E[\widetilde{R}^{eff}] = 1$. Further, by Lemma 2, the total expected marginal costs of deposit and equity financing must equal the expected return on bank loans that, in turn, equals the expected marginal return on firm investment. We, thereby, obtain (48). As depositors invest all their capital in banks, we have $D^{reg} = \mathcal{D}$. The equity capital, E^{reg}, invested in banks is pinned down by the condition that the total inestment in the safe asset must match the efficient investment level, X_S^{eff}. As the proportion, β^{reg}, of total bank equity plus deposit capital, and the capital $\pi^{reg} D^{reg}$, in the deposit insurance fund is invested in the safe asset, we obtain the second condition in (49).

Incomplete Insurance for Depositors

By Theorem 2, the efficient allocation features incomplete insurance for depositors if $\tau > \overline{\tau}$.

Theorem 4 (Implementation of Efficient Allocation - Incomplete Insurance) *1. There exists a continuum of policy tools all of which implement the efficient allocation. For each (β^{reg}, π^{reg}) satisfying (45), there exists a unique corresponding capital requirement and tax-financed bailout policy defined by the marginal tax rates on deposit payments by successful banks; the marginal subsidies (subsidies per unit of deposits) to failed banks; the expected marginal taxes on bank equity payments; the expected marginal taxes on firm equity payments; and the expected lump sum taxes on entrepreneurs; that implement the efficient allocation.that implement the efficient allocation. For a given set of policy tools, the equilibrium of the regulated economy is unique.*

2. The returns to bank depositors and equity holders, the expected marginal costs of deposit and *equity financing for banks, and the loan rate in the regulated equilibrium are as follows.*

$$R_D^{j,reg} = \frac{P_D^{j,eff}}{\mathcal{D}} \text{ for } j \in \{H, L\};\ R_D^{H,reg} > 1 > R_D^{L,reg};\ E\left[\widetilde{R}_D^{reg}\right] > 1 \tag{51}$$

$$R_E^{f,j,reg} = 0 \text{ for } j \in \{L, H\},\ E[\widetilde{R}_E^{reg}] = \frac{\Delta_E}{p} > 1 \tag{52}$$

$$E[\tilde{C}_E^{reg}] = qR_L^{reg} = E[\widetilde{R}^{eff}] > 1, \tag{53}$$

$$E[\tilde{C}_D^{reg}] = E[\widetilde{R}^{eff}] - \frac{X_S^{eff} - \pi^{reg}\mathcal{D}}{\mathcal{D}}(E[\widetilde{R}^{eff}] - 1) < E[\tilde{C}_E^{reg}], \tag{54}$$

The expected marginal cost of bank equity is strictly greater than the expected marginal cost of bank deposits.

3. *The total amounts of equity capital, deposit capital, loan capital and investments in the safe asset in the regulated equilibrium are*

$$D^{reg} = \mathcal{D},\ E^{reg} = \frac{X_S^{eff} - \pi^{reg}D^{reg} - \beta^{reg}D^{reg}}{\beta^{reg}} \tag{55}$$

$$L^{reg} = (1 - \beta^{reg})(E^{reg} + D^{reg}),\ S^{reg} = X_S^{eff} - \pi^{reg}D^{reg}. \tag{56}$$

Similar to the case when $\tau \leq \bar{\tau}$, there is a continuum of regulatory policies that implement the efficient allocation, but the equilibrium of the economy is unique for a given regulatory policy. In contrast with the earlier case, however, *all four* components of the regulatory policy are *essential* to implement the efficient allocation. In other words, capital and liquidity requirements are not just required to ensure that the efficient allocation is uniquely implemented, but are necessary to implement the allocation along with deposit insurance and bailouts. The intuition hinges on the fact that, if $\tau > \bar{\tau}$, the equilibrium expected marginal return on investment in firms and, therefore, the expected return on bank loans is greater than one (see (33) and (52)). Hence, banks do not voluntarily invest in the safe asset. The liquidity requirement is, therefore, necessary to force banks to invest in the safe asset. For the reasons described earlier, the capital requirement ensures that bank size is sufficient to ensure that the economy in the aggregate invests the optimal amounts of capital in firms and the safe asset.

As the expected return on bank loans exceeds one, it follows from Lemma 2 and (43) that the expected return on bank loans equals the expected marginal cost of equity financing. Since the liquidity requirement is binding, however, the expected return on a bank's assets is strictly lower than the expected loan return. Because banks make zero expected profits in equilibrium, the expected marginal cost of bank deposits must be strictly less than the expected marginal cost of bank equity. The binding liquidity requirement, therefore, creates a positive wedge between the expected marginal costs of equity and deposits for banks.

Partial equilibrium models of banking typically exogenously assume that the cost of deposit financing is lower than the cost of equity financing. We derive this prediction endogenously in our general equilibrium framework. Theorems 3 and 4, however, show that the expected marginal cost of deposit financing is lower than that of equity financing *only when* aggregate risk exceeds a threshold so that depositors are imperfectly insured, and liquidity and capital requirements are binding. If the regulated equilibrium features full insurance for depositors, the expected marginal costs of deposit and equity financing are equal in equilibrium as shown by Theorem 3.

6.4 Properties of Optimal Regulatory Policies

In this section, we derive properties of the optimal regulatory policies and explore further comparative statics relationships.

Proposition 4 (Tax and Bailout Policies) *1. An equilibrium with a lower or more lax liquidity requirement, β^{reg}, is associated with a stricter capital requirement, θ^{reg}, and a larger representative bank size.*

2. The tax and bailout policy is as follows.

(a) If $0 < \tau < \overline{\tau}$

$$t_D^{H.reg} = t_D^{L.reg} < 0,\ E[\widetilde{t}_D^{reg}] < 0,\ \phi^{L,reg} > \phi^{H,reg} \tag{57}$$

$$E[\widetilde{t}_E^{reg}] < 0,\ E[\widetilde{T}_F^{reg}] > 0. \tag{58}$$

(b) If $\overline{\tau} < \tau \leq \hat{\tau}$, where $\widehat{\tau}$ is defined in Proposition 3, then

$$t_D^{H.reg} \neq t_D^{L.reg};\ E[\widetilde{t}_D^{reg}] \leq 0,\ \phi^{L,reg} > \phi^{H,reg}, \tag{59}$$

$$E[\widetilde{t}_E^{reg}] \leq 0, E[\widetilde{T}_F^{reg}] \geq 0 \tag{60}$$

(c) If τ¿$\hat{\tau}$, *then*

$$t_D^{H.reg} \neq t_D^{L.reg};\ E[\widetilde{t_D^{reg}}] > 0,\ \phi^{L,reg} > \phi^{H,reg}, \tag{61}$$

$$E[\widetilde{t_E^{reg}}] > 0, E[\widetilde{T_F^{reg}}] < 0 \tag{62}$$

The regulator must ensure that aggregate investments in firms and the safe asset by the economy as a whole are efficient. The aggregate allocation does not, however, pin down bank size. We can have regulated equilibria with differing bank sizes (or sizes of the financial sector), but still have the aggregate allocation to production and the safe asset be efficient. Because depositors invest all their capital in banks, bank size is determined by the amount of equity capital that banks raise. An equilibrium with larger banks implies that equityholders invest more capital in banks than directly in firms. As the liquidity requirement stipulates that banks invest a minimum *proportion* of their capital in the safe asset, an equilibrium with larger banks is associated with a *lower* liquidity requirement to guarantee that the aggregate investment in the safe asset corresponds to the efficient allocation. A regulated equilibrium with larger banks is also associated with a *stricter* capital requirement to ensure that banks raise more equity capital.

Part 2 of the proposition shows that the tax and bailout policies depend crucially on the level of aggregate risk. Let us consider each interval of aggregate risk in turn.

When $\tau < \overline{\tau}$, the efficient allocation features full insurance for depositors. Hence, the marginal tax rates on deposit payments by successful banks are the same in both aggregate states as shown by 57. Depositors and equityholders receive subsidies in expectation, while entrepreneurs pay taxes. The intuition is as follows. Recall from Proposition 3 that, when $\tau < \overline{\tau}$, the unregulated economy underinvests relative to the efficient allocation. Hence, the regulator's objective is to increase the aggregate investment in productive firms. If the aggregate investment in firms were at the efficient level, therefore, the the total payoff of entrepreneurs would exceed their total payoff, Δ_F, in the unregulated equilibrium in the absence of any taxes or subsidies. Because the efficient allocation maximizes the expected utility of depositors, while maintaing the payoffs of entrepreneurs and equityholders at their unregulated equilibrium levels, there must be positive taxes imposed on entrepreneurs in expectation to bring their payoffs down to the unregulated equilibrium level. At the same time, the presence of decreasing returns to scale in production implies that the expected marginal return on investment in the efficient allocation is lower than in the unregulated equilibrium. Part 2 of Theorem 3 shows that the expected marginal costs of equity and deposit financing for banks are equal to one, while the expected returns to depositors and equityholders are greater than one. Hence, by (40) and (41), in expectation, depositors and equityholders must receive subsidies that are financed by lump sum taxes on entrepreneurs.

When $\tau > \overline{\tau}$, depositors bear aggregate risk in the efficient allocation so the marginal tax rates on depositors are no longer equal in the high and low aggregate states. If $\overline{\tau} < \tau \leq \hat{\tau}$, depositors and equityholders receive subsidies in expectation that are financed by lump sum taxes on firms as in Proposition 4. If $\tau¿\hat{\tau}$, however, the reverse is true with depositors and equityholders paying taxes in expectation with firms receiving subsidies. The intuition is as follows.

By Proposition 3, the unregulated economy underinvests in firms when $\tau < \widehat{\tau}$, but overinvests when $\tau > \widehat{\tau}$. When $\tau < \hat{\tau}$, depositors and equityholders receive subsidies in expectation, while entrepreneurs pay taxes for the same reasons discussed above 4 that explain the policy in the interval $\tau < \overline{\tau}$ However, when $\tau > \hat{\tau}$, the unregulated economy overinvests in production. Consequently, if the aggregate investment in firms is at the efficient level, the total payoff of entrepreneurs in the absence of any taxes or subsidies would be lower than their total payoff, Δ_F, in the unregulated equilibrium. To implement the efficient allocation, therefore, the regulator must increase the expected payoff of entrepreneurs to the unregulated equiibrium level, Δ_F by providing subsidies to entrepreneurs. Let us now consider equityholders. Since the unregulated economy overinvests, the regulator must lower the level of investment in firms. The presence of decreasing returns to scale in production implies that the expected marginal return on investment is greater than the expected marginal return in the unregulated economy. In the absence of any taxes or subsidies on equityholders, the expected return to firm equityholders equals the expected marginal return on investment in firms. Because equityholders must be indifferent between investing in bank and firm equity, the expected return to bank equityholders also equals the expected marginal return on investment in firms. Hence, the expected payoff to equityholders exceeds the expected payoff, Δ_E, in the unregulated equilibrium. As the expected payoff of equityholders is Δ_E in the efficent allocation, they must pay taxes in expectation to ensure that their expected payoff in the regulated equilibrium is Δ_E. Finally, let us consider depositors. As discussed above, the unregulated economy overinvests in production for $\tau > \widehat{\tau}$. In the absence of taxes or subsidies, therefore, the *combined expected payoffs* of entrepreneurs and equityholders would lower than their combined payoff, $\Delta_E + \Delta_F$, in the unregulated equilibrium. To implement the efficient allocation, therefore, the regulator must increase the combined expected payoff of entrepreneurs and equityholders. Because the regulator must maintain a balanced budget, this can only be financed by levying taxes in expectation on depositors.

The above results are interesting in the context of the ongoing debate on bank regulation. Some commentators have proposed significantly reducing, or even eliminating, the tax deductibility of bank debt interest payments to reduce banks' incentives to take on excessive leverage (e.g., see Admati and Hellwig (2013)). The above results suggest that, when aggregate risk is below a threshold, depositor subsidies are, in fact, be efficient to mitigate underinvestment and achieve optimal risk-sharing among depositors, equityholders and entrepreneurs in a general equilibrium setting. Subsidizing bank debt via tax shields effectively achieves the same objective. However, when aggregate risk is above a threshold, depositor subsidies are no longer efficient. Proposition 4 further shows that, when aggregate risk is below a threshold, it is, in fact, also efficient to provide subsidies to bank and firm equityholders with depositor and investor subsidies being financed by taxation on entrepreneurs. However when aggregate risk is above the threshold, it is optimal to subsidize entrepreneurs and levy taxes in expectation on depositors and equityholders.

The following proposition describes relations among the different regulatory tools.

Proposition 5 (Relations Among Regulatory Tools)

1. *The representative bank's expected marginal cost of equity financing, and the representative firm's expected marginal costs of bank loan and equity financing are independent of regulatory policy tools: the liquidity requirement, capital requirement, deposit insurance and taxes. The expected marginal cost of deposit financing only depends on the deposit insurance policy.*

2. *The capital requirement, θ^{reg} decreases with the liquidity requirement, β^{reg} and increases with the deposit insurance premium, π^{reg}.*

3. *The deposit insurance subsidy (per unit of deposits); marginal taxes on depositors and equityholders in the high aggregate state; and lump sum taxes on entrepreneurs in the high aggregate state all increase in the liquidity requirement, β^{reg} and decreases with the deposit insurance premium, π^{reg}.*

4. *The marginal tax rate on on equity holders and lump sum taxes on entrepreneurs in the low*

aggregate state decrease with the liquidity requirement, β^{reg}.

By (37), the liquidity requirement, β^{reg}, determines the proportion of a bank's total capital that must be invested in the safe asset to guarantee that the economy as a whole allocates capital efficiently to productive firms and the safe asset. The right endpoint of the interval of possible values of β^{reg} in (45) (for a given deposit insurance premium, π^{reg}) corresponds to a regulated equilibrium in which banks are financed solely via deposits. The left endpoint corresponds to a regulated equilibrium in which banks raise all available equity capital, $\mathcal{E}$. The capital requirements in the "full insurance for depositors" and "incomplete insurance for depositors" cases, respectively, ensures that banks raise the requisite amount of equity capital to guarantee that the economy as a whole invests the efficient amount of capital in firms. The deposit insurance and bailout policies then ensure that the payoffs of depositors, equity holders and entrepreneurs correspond to the efficient allocation.

As we discussed earlier, in any regulated equilibrium, equityholders must be indifferent between investing in banks and firms, and firms must be indifferent between bank loan and equity financing. Further, the expected return on bank loans must equal the expected marginal return on investment in firms. The expected marginal return on investment in firms is determined entirely by the amount of capital that the economy in the aggregate invests in firms. Because equilibria differ only in the relative proportions of capital that equity holders provide to banks vis-a-vis firms, the aggregate allocations of capital to firms and the safe asset are the same across these equilibria. Hence, the expected marginal cost of bank equity financing, and the expected marginal costs of bank loan and equity financing for firms are identical across the equilibria and independent of regulatory policy tools. By Theorems 3 and 4, teh expected marginal cost of bank deposit financing is strictly lower than the expected marginal cost of bank equity financing only when $\tau > \overline{\tau}$ so that depositors bear aggregate risk in the efficient allocation. The wedge between the expected marginal costs of bank equity and deposit financing depends only on the amount of deposit insurance as determined by the deposit insurance premium.

Part 2 re-states our earlier result that a *stricter* liquidity requirement (higher β^{reg}) is associated with a *lower* capital requirement (lower θ^{reg}). A stricter liquidity requirement implies that banks must invest a greater proportion of their total capital in the safe asset. To ensure that the efficient amount of total capital in the economy is invested in firms, banks must, therefore, raise less equity. In other words, a greater proportion of the capital held by equity holders goes to firms rather than banks. Because banks are smaller as the liquidity requirement becomes stricter, this also means that firms are financed through a greater proportion of outside equity capital relative to bank loans. Due to the substitutable roles of deposit insurance and the liquidity requirement, they havec opposing effects on the capital requirement. Since the capital requirement decreases with the liquidity requirement, it increases with the deposit insurance premium.

Equity holders, banks and firms share the total surplus net of payments to depositors. Hence their total payoff decreases in the high aggregate state, and increases in the low aggregate state as β^{reg} increases. As the total surplus does not vary with β^{reg} in each state (the equilibria corresponding to different β^{reg} all implement the same efficient allocation), it follows that total taxes on equity holders, banks and firms decrease (increase) in the high (low) aggregate state.

The following is a corollary of Proposition 5.

Corollary 1 *There exists a a threshold $\bar{\beta} \in [0, 1)$ such that the marginal tax rate on depositors of successful banks is positive for $\beta^{reg} < \bar{\beta}$ and negative for $\beta^{reg} > \bar{\beta}$.*

As β^{reg} increases, a higher portion of the representative bank's capital is invested in the safe asset, which lowers its expected asset return, thereby lowering its expected payment to depositors (see (44)). When β^{reg} exceeds a threshold, the expected payment by the bank to depositors declines sufficiently that even depositors of successful banks must receive subsidies to ensure that their payoff allocation is efficient. The above corollary further highlights the point we discussed earlier that subsidies to depositors of successful banks may, in fact, be efficient. The result shows that the nature of depositor subsidies is intimately tied to the liquidity requirement.

At a broad level, Theorems 3 and 4 provide some support for both proponents and opponents of strict bank regulation. There is, in fact, a range of regulatory policies all of which implement the *same* efficient allocation. The regulated equilibria generated by the range of policies differ in the tightness of the liquidity constraint and the resulting size of banks. Importantly, however, the different regulatory tools must be tuned to each other to ensure that the regulatory policy actually implements the efficient allocation. In particular, a *stricter* liquidity requirement is associated with a *looser* capital requirement and less deposit insurance.

We now investigate how aggregate risk influences the optimal regulatory policy.

Theorem 5 (Aggregate Risk and Regulatory Policy)

- *If $\tau \leq \bar{\tau}$, the optimal regulatory policy is independent of τ.*
- *If $\tau > \bar{\tau}$, as τ increases,*
  - *the interval of possible values of the liquidity parameter, β^{reg}, described in Theorems (3) and (4) shifts to the right (keeping the deposit insurance policy determined by the premium, π^{reg} fixed);*
  - *the expected before-tax bank deposit return as well as the before-tax bank and firm equity returns all increase;*
  - *for each β^{reg}, the corresponding capital requirement becomes tighter;*
  - *the aggregate risk, τ, has ambiguous effects on the deposit insurance and taxation policies.*

By Theorem 2, the efficient allocation does not vary with τ for $\tau \leq \overline{\tau}$. Hence, the optimal regulatory policy is unaffected by aggregate risk in this region. When $\tau > \overline{\tau}$, however, the efficient level of investment in the safe asset increases. Consequently, for a given bank capital level, the liquidity requirement is stricter. Further, for a *given* value of β^{reg} that can implement the efficient allocation, the total amount of capital invested in the safe asset must increase as τ increases. As a result, banks must raise more equity capital so the capital requirement becomes stricter. Because the efficient level of investment in entrepreneurial firms decreases with τ, the concavity of firms' production technology implies that the expected marginal return on investment in firms increases. As a result, the expected return on bank loans, firm equity, bank equity and bank deposits all increase.

7 Conclusions

We develop a tractable general equilibrium model of competitive banks that are exposed to sectoral and aggregate shocks. There is a continuum of unregulated equilibria that vary from an equilibrium in which banks are financed purely with debt and the real economy is financed with bank debt and equity, to a "full intermediation" equilibrium in which banks raise all available equity capital and the real economy is financed entirely via bank debt. The unregulated economy underinvests in production when aggregate risk is below a threshold, but overinvests when aggregate risk is above the threshold. We characterize the efficient allocations and demonstrate how regulatory intervention tools can be used to implement the efficient allocations in a decentralized economy.

For a given efficient allocation, there is a range of regulatory policies all of which implement the allocation, but the various tools in an optimal regulatory policy must be finely tuned to each other. In particular, capital and liquidity requirements move in opposing directions; an optimal regulatory policy that features a stricter capital requirement has a looser liquidity requirement. When aggregate risk is below a threshold, the efficient allocation can be implemented via deposit insurance and taxation alone, but capital and liquidity requirements are necessary to ensure that the equilibrium of the regulated economy is unique. When aggregate risk is high, all four regulatory tools are essential components of an optimal regulatory policy.

Our results provide qualified support for proponents and opponents of stricter banking regulation. Lower capital requirements for banks could be optimal, but they must be accompanied by stricter liquidity requirements and vice versa. The range of optimal capital and liquidity requirements are unaffected by aggregate risk when it is low, but become tighter as aggregate risk increases above the threshold. It is efficient for depositors to receive subsidies in expectation when aggregate risk is below a threshold, but depositors should pay taxes in expectation when aggregate risk is above the threshold.Our results shed some light on the debate generated by proposals to reduce, or even eliminate, the tax deductibility of bank debt interest payments to lower banks' incentives to take on excessive leverage. We argue that depositor subsidies (that can be implemented via debt tax shields) are, in fact, efficient, but only if aggregate risk is below a threshold. General equilibrium effects and, in particular, the roles of bank debt and equity as well as taxation in achieving risk-sharing between depositors and equityholders play central roles in generating our results.

The basic framework we develop can be extended in several ways. We could extend the model to a dynamic setting that would allow us to address how bank regulation varies with fluctuations in the business cycle. A second potential extension would be to introduce liquidity shocks among depositors and/or among banks' assets, thereby creating the possibility of bank runs. A third possible extension would be to introduce Keynesian frictions, thereby creating a role for monetary policy. We leave the analyses of these and other extensions to future research.

Appendix

Proof of Theorem 1

First we show that in any unregulated equilibrium that banks go bankrupt, all the equations stated in the theorem are satisfied. Lemma 1 states that banks and equity holders do not invest in the safe asset. Equalizing marginal cost and marginal revenue in the entrepreneurs' problem and the fact that equity holders invest all of their capital (directly or indirectly) in the risky asset imply equation (14). Equation (15) follows from the accounting equality. Lemma 1 implies equation (16). Equation (17) is satisfied by the definition of the supply function $d(R_1, R_2)$. Equations (18) and the equality $R_E^{f,unreg} = 0$ are satisfied because banks go bankrupt in the risky deposit equilibrium. When the representative bank's pool of loans fails, banks cannot fulfill the promised rate R_D^{unreg}. Hence, the bank's total payoff, which is equal to $\omega_L R_L^{unreg} L^{unreg}$, goes to depositors. Equation (19) follows from the fact that equity holders are residual claimants of the banks and do not get anything when the bank defaults.

We now show that the prices and quantities defined in the statement of the Theorem constitute an unregulated equilibrium. Equality $E[\tilde{R}_E^{unreg}] = qR_L^{unreg}$ implies that equity holders are indifferent between investing in firms and banks. Hence, equity holders would supply the proposed level of equity stated in the theorem. Equation (17) implies that depositors, given the deposit rates, supply the optimal level of capital. Equations (14) and (15) imply that the loan, equity and deposits market clear. Finally, we show banks make zero profit in the proposed equilibrium. By equations (18) and (19) we can see that the total revenue of the banks which is $\omega_L R_L^{unreg} L^{unreg}$ in the low state and $\omega_H R_L^{unreg} L^{unreg}$ in the high state, is exactly equal to the total payouts to depositors and equity holders in both states, so banks make zero profit. Moreover, banks cannot make a positive profit since the expected marginal cost of deposits and equity is equal to the bank's expected investment return.

Proof of Proposition 1

For a given level of deposits D^{unreg}, we want to find conditions under which a risky deposits equilibrium exist in which depositors supply D^{unreg}. This is equivalent to finding a solution for equations in Theorem 1 where depositors supply D^{unreg}. Since $\Lambda'(\frac{X^{unreg}}{\mathcal{F}}) = R_L^{unreg}$ and Λ is concave, given D^{unreg}, R_L^{unreg} is uniquely identified; and R_L^{unreg} is a decreasing function of D^{unreg}. The discussion following Proposition (1) shows that given a level of deposits D^{unreg} that satisfies $\Lambda'(\frac{D^{unreg}+\mathcal{E}}{\mathcal{F}}) > \frac{1}{q}$, the problem reduces to solving the following system of equations:

$$E[\tilde{R}_D^{unreg}] = pR_D^{s,unreg} + (1-p)R_D^{f,unreg} = q\Lambda'(\frac{D^{unreg} + \mathcal{E}}{\mathcal{F}}) > 1,$$
$$D^{unreg} = \mathcal{D}d(\tilde{R}_D^{unreg}) = \mathcal{D}d(R_D^{s,unreg}, R_D^{f,unreg}).$$

We identify the bounds on D^{unreg} such that the system above has a solution. Since depositors have a continuous utility function and are risk averse, when $E[\tilde{R}_D^{unreg}]$ is fixed, the supply of deposit is continuously decreasing in $R_D^{s,unreg} - R_D^{f,unreg}$. In addition, $R_D^{s,unreg} - R_D^{f,unreg}$ is minimized when $E^{unreg} = \mathcal{E}$ so that banks raise all available equity capital. Set $R_D^{s,unreg}(D)$ and $R_D^{f,unreg}(D)$ to be the deposit rates when banks raise all available equity, raise D in deposits and the expected deposit rate is $q\Lambda'(\frac{D+\mathcal{E}}{\mathcal{F}})$. The upper bound on the deposits, D^{unreg} is the largest solution to the following:

$$D_{max} = \mathcal{D}d(R_D^{unreg}(D_{max}), R_D^{f,unreg}(D_{max})). \tag{63}$$

Equations (63) correspond to an unregulated economy where banks raise all of available equity in the market. Equation (63) has a nontrivial interior solution because the R.H.S. is strictly positive if $D_{max} = 0$ as the condition that implies that depositors provide nonzero capital to banks. The R.H.S. equals zero for $D_{max} = \mathcal{D}$ because the condition, $q\Lambda'(\frac{\mathcal{D}+\mathcal{E}}{\mathcal{F}}) < 1$, implies that depositors do not invest in banks. Hence, by continuity, there exists at least one non-trivial interior solution for (63). For the lower bound on deposits, note that we have $R_D^{f,unreg} = \frac{\omega_L R_L^{unreg} L^{unreg}}{D^{unreg}} \geq \omega_L R_L^{unreg}$. The supply of deposits is minimized if there is equality, which is equivalent to $E^{unreg} = 0$ and $R_D^{unreg} = \omega_H R_L^{unreg}$. To find D_{min} then it is enough to solve the following equation

$$D^{unreg} = \mathcal{D}d(\tilde{R}_D^{unreg}) = \mathcal{D}d(\omega_H R_L^{unreg}, \omega_L R_L^{unreg}) \tag{64}$$

where R_L is given by $\Lambda'(\frac{X^{unreg}}{\mathcal{F}}) = \Lambda'(\frac{\mathcal{E}+D^{unreg}}{\mathcal{F}})$. Equations (64) corresponds to an unregulated economy where banks raise no equity. Again like the D_{max} case, using continuity, we can see there is interior solution for equation (64).

Note that when D^{unreg} increases $R_D^{f,unreg}$ also increases. Then equation (18) implies that L^{unreg} increase as well which completes the argument.

Proof of Theorem 2

To simplify the problem, set

$$\Delta_E + \Delta_F = \Delta$$
$$P_E^{\omega} + P_F^{\omega} = P_N^{\omega}$$

Now if in the central planner problem we get $E[P_N^{\omega}] \geq \Delta$, using a cash transfer between firms and equity holders we can get both

$$E[P_E^{\omega}] \geq \Delta_E; E[P_F^{\omega}] \geq \Delta_F.$$

so it is enough to solve the problem for Δ and P_N^ω. More precisely the problem reduces to

$$\max_{X_F, X_S, P_D^L, P_D^H, P_E^L, P_E^H \geq 0} E[u(P_D^\omega)] \text{ subject to}$$

$$X_S + X_F \leq \mathcal{D} + \mathcal{E} \tag{65}$$

$$P_D^H + P_N^H \leq \gamma_H \mathcal{F} \Lambda(\frac{X_F}{\mathcal{F}}) + X_S \tag{66}$$

$$P_D^L + P_N^L \leq \gamma_L \mathcal{F} \Lambda(\frac{X_F}{\mathcal{F}}) + X_S, \tag{67}$$

$$E[P_N^\omega] \geq \Delta \tag{68}$$

In particular, by limited liability we have

$$P_D^H \leq \gamma_H \mathcal{F} \Lambda(\frac{X_F}{\mathcal{F}}) + X_S = A_1 \tag{69}$$

$$P_D^L \leq \gamma_L \mathcal{F} \Lambda(\frac{X_F}{\mathcal{F}}) + X_S = A_2 \tag{70}$$

and for expected payout to depositors we have

$$E[P_D^\omega] \leq pA_1 + (1-p)A_2 - \Delta = q\mathcal{F}\Lambda(\frac{X_F}{\mathcal{F}}) + X_S - \Delta = A_0 \tag{71}$$

where in above we used the fact that $p\gamma_H + (1-p)\gamma_L = q$. First note that in the optimum, all three equations (65),(66) and (67) are binding. This is because central planner will not dispose any capital and will distribute it fully among agents. Now we show that in the central planner problem the equation (71) is also binding. Here is the reason. If not, then we have $E[P_N^\omega] > \Delta$ and since we have

$$A_0 < pA_1 + (1-p)A_2$$

at least one of equations (66) or (67) is not binding. So we can increase payout to depositors in at least one of the states without violating equation (68) which contradicts optimality and proves our claim. Now we consider two cases.

- Suppose equation (68) is binding, but the equation (70) is not binding. In this case, since depositors are risk averse, we should have

$$P_D^H = P_D^L = \max_{X_F, X_S} q\mathcal{F}\Lambda(\frac{X_F}{\mathcal{F}}) + X_S - \Delta = A_0$$

This is because otherwise we can transfer money from high state to low state such that equation $E[P_D^\omega] = A_0$ is satisfied. By risk aversion when expected payout is fixed, agents prefer smoother payout so utility will increase which proves our claim. Now recall that since central planner optimally invest all the resources in either safe asset or firms we have $X_S = \mathcal{D} + \mathcal{E} - X_F$. Getting first order condition of the above equation gives us

$$\Lambda'(\frac{X_F^{eff}}{\mathcal{F}}) = \frac{1}{q}$$

which corresponds to the full insurance case. This can happen only when there is enough money to payout to depositors in the low state so we should have

$$A_0 \leq A_2$$

which is equivalent to

$$\tau \leq \frac{\Delta}{p\mathcal{F}\Lambda(\frac{X_F^{eff}}{\mathcal{F}})(\omega_H - \omega_L)} = \bar{\tau}$$

as we claimed.

- Now suppose the equation (70) is binding in addition to equation (68) . Then we have

$$P_D^H = A_1 - \frac{\Delta}{p}$$

since $E[P_D^\omega] = A_0 = pA_1 + (1-p)A_2 - \Delta$. So the central planner's problem which is

$$\max_{X_F}[pu(P_D^H) + (1-p)u(P_D^L)]$$

and gives us equation (32) as claimed by the explicit expression of P_D^H and P_D^L.

Proof of Proposition 2

For $\tau < \bar{\tau}$, as stated in equation (29), X_F^{eff} is constant. For $\tau > \bar{\tau}$, in order to prove X_F^{eff} is decreasing, it is enough to show the cross derivative of the objective function is negative. The cross derivative is:

$$\begin{aligned}
&\frac{\partial^2}{\partial X_F \partial \tau}[pu(P_D^H) + (1-p)u(P_D^L)] = \\
&p(1-p)(\omega_H - \omega_L)\Lambda'(\frac{X_F}{\mathcal{F}})(u'(P_D^H) - u'(P_D^L)) + \\
&p(1-p)(\omega_H - \omega_L)\mathcal{F}\Lambda(\frac{X_F}{\mathcal{F}})[(\gamma_H\Lambda'(\frac{X_F}{\mathcal{F}}) - 1)u''(P_D^H) - (\gamma_L\Lambda'(\frac{X_F}{\mathcal{F}}) - 1)u''(P_D^L)]
\end{aligned}$$

Concavity of the utility function u implies that the first term is negative.. It is enough to show the last term is negative as well. It suffices to show the following:

$$(\gamma_H\Lambda'(\frac{X_F}{\mathcal{F}}) - 1)u''(P_D^H) - (\gamma_L\Lambda'(\frac{X_F}{\mathcal{F}}) - 1)u''(P_D^L) < 0$$

Since $u'' < 0$, it is enough to show that

$$A^H = \gamma_H \Lambda'(\frac{X_F}{\mathcal{F}}) - 1 \geq 0$$
$$A^L = \gamma_L \Lambda'(\frac{X_F}{\mathcal{F}}) - 1 \leq 0$$

The first order condition for X_F in the central planner problem is (equation (32)):

$$pA^H u'(P_D^H) + (1-p)A^L u'(P_D^L) = 0.$$

Since $u'(P_D^H), u'(P_D^L) > 0$, A^H and A^L must have opposite signs. Note that:

$$A^H - A^L = (\gamma_H - \gamma_L)\Lambda'(\frac{X_F}{\mathcal{F}}) > 0.$$

Hence $A^H > 0 > A^L$. This completes the proof.

Proofs of Theorems 3 and 4

Note that the allocation stated in the theorem is efficient. With the policy tools stated in the theorem depositors supply all of their capital to banks[4], equity holders are indifferent between banks and firms, and banks make zero profit. Therefore, the proposed decisions constitute an equilibrium. Given the policy tools, we now show that the equilibrium is unique.

Note that when a bank's pool of loan fails, the total cash flow is

$$\omega_L R_L^{reg} L^{reg} + X_S^{eff}.$$

When the representative bank fails, its entire payoff goes to depositors. Therefore

$$\mathcal{D} C_D^{f,reg} = \omega_L R_L^{reg} L^{reg} + X_S^{eff}, \tag{72}$$

which gives us the equation for $C_D^{f,reg}$ as in the theorem. To calculate $E[\tilde{C}_D^{reg}]$, we have:

$$E[\tilde{C}_D^{reg}] = pC_D^{s,reg} + (1-p)C_D^{f,reg} = E[R^{eff}] - \frac{X_S^{eff}}{\mathcal{D}}(E[R^{eff}] - 1).$$

Since this is less than (proposed) value for $E[\tilde{R}_E^{reg}] = E[R^{eff}]$, banks prefer deposits to equity and hence they attract all the available deposit. This shows the deposit market clears. Note that banks do not pay anything to equity holders when their pool of loans fail. When the banks pool of loan succeed, the return to equity is followed from:

$$R_E^{reg} E^{reg} + R_D^{s,reg} \mathcal{D} = \omega_H R_H^{reg} L^{reg} + X_S^{eff},$$

[4] Since $P_D^{H,eff} \geq P_D^{L,eff} \geq \mathcal{D}$ deposits are at least as good as the safe asset in every state, hence depositors invest all of their capital in the banks.

where we equalize the revenue of banks with the payout to depositors and equity holders. Therefore,

$$R_E^{reg} = \frac{1}{E^{reg}}[\omega_H R_H^{reg} L^{reg} + X_S^{eff} - R_D^{s,reg}\mathcal{D}]$$

If we set $E^{reg} = \frac{X_S^{eff} - \beta^{reg}\mathcal{D}}{\beta^{reg}}$ and $L^{reg} = (1-\beta^{reg})(E^{reg} + \mathcal{D})$ we get $pR_E^{reg} = qR_L^{reg}$. Parameter θ^{reg} is defined using the (binding) relation (38) which is in this case

$$\mathcal{D} = (1-\theta^{reg})L^{reg} + X_S^{eff}.$$

This gives us $\theta^{reg} = 1 - \frac{\beta^{reg}(\mathcal{D} - X_S^{eff})}{X_S^{eff}(1-\beta^{reg})}$. Since equity holders are indifferent between investing in firms and banks, we can assume they invest in the banks as much as banks demand in the equilibrium. In order to show banks demand $E^{reg} = \frac{X_S^{eff} - \beta^{reg}\mathcal{D}}{\beta^{reg}}$ in equity capital, we have to verify equation (42) which is

$$\beta^{reg} + (1-\beta^{reg} - \alpha^{reg})qR_L^{reg} = (1-\alpha^{reg})E[\tilde{R}_D^{reg}]$$

where $\alpha^{reg} = (1-\theta^{reg})(1-\beta^{reg})$. Substituting $E[\tilde{R}_D^{reg}]$ from above we should show that $\beta^{reg} = (1-\alpha^{reg})\frac{X_S^{eff}}{\mathcal{D}}$, which is equivalent to showing that

$$\frac{\beta^{reg}(\mathcal{D} - X_S^{eff})}{X_S^{eff}} = \theta^{reg}(1-\beta^{reg}),$$

but the above holds by the definition of θ^{reg}. Finally, firms demand exactly X_F^{eff} given R_L^{eff} because $R_L^{eff} = \frac{E[R^{eff}]}{q} = \Lambda'(\frac{X_F^{eff}}{\mathcal{F}})$. We have thus established that the prices and quantitities constitute an equilibrium. We now establish that the equilibrium is unique.

Note that the capital and liquidity requirements mandate that banks to raise at least E^{reg} in equity and invest at least S^{reg} in the safe asset. If the capital or liquidity requirements are not binding then the expected cost of deposits and equity must be equal, $E[C_D^{reg}] = E[C_E^{reg}]$, since banks invest a non zero portion of their capital in the safe asset. Lemma 2 then implies that $qR_L^{reg} = E[C_D^{reg}] = E[C_E^{reg}] = 1$. Note that if the capital or liquidity constraints are not binding, then banks invest more in the safe asset which implies that $1 = qR_L^{reg} > E[R^{eff}]$, which is a contradiction.

If depositors invest their entire capital in banks, $D^{reg} = \mathcal{D}$. Lemma 2 uniquely identifies the return to equity, deposits and loans. Once the capital structure of the banks is determined, the rest of the equilibrium allocations follow uniquely. This argument implies that it is enough to show that in any equilibrium, $D^{reg} = \mathcal{D}$. We now show this.

We claim that if $D^{reg} < \mathcal{D}$, then $qR_L^{reg} > E[\tilde{R}^{eff}]$. Note that, if banks raise D^{reg}, then they have to raise at least $E^{reg} = \theta L^{reg}$ in equity, because of the capital requirement. Hence, the total investment in firms is at most:

$$(1-\beta)(\theta L^{reg} + D^{reg}) + \mathcal{E} - E^{reg}$$

The relation $L^{reg} = (1-\beta)(E^{reg} + D^{reg})$ and $E^{reg} = \theta L^{reg}$, implies

$$E^{reg} = \theta \frac{1-\beta}{1-(1-\beta)\theta} D^{reg} = \gamma D^{reg}$$

Hence, the total investment in firms is at most

$$[(1-\beta)(1+\gamma) - \gamma] D^{reg} + \mathcal{E}.$$

To show that $D^{reg} = \mathcal{D}$, we need to show $(1-\beta)(1+\gamma) - \gamma > 0$. This is equivalent to showing that

$$\beta(1-\beta)\theta < (1-\beta)[1-(1-\beta)\theta],$$

but the above is true if $\theta < 1$, which is obvious. Hence, we have

$$qR_L^{reg} > E[\tilde{R}^{eff}]. \tag{73}$$

Inequality (73) implies that banks' payment to depositors is higher in the low state compared to the equilibrium allocation stated in the theorem. However, once we implement the tax policy, the return on deposit becomes at least as good as safe asset (as in the proposed equilibrium). Hence, depositors invest all of their capital in banks which shows that $D^{reg} < \mathcal{D}$ cannot be true in equilibrium.

The proof of Theorem 3 follows using arguments that are only minor modifications of the above arguments. We, therefore, omit it for brevity.

Proof of Proposition 5

As mentioned in the text, the first part of the proposition is the corollary of the fact that these values are determined by the invested capital X_F^{eff}, which is independent of policy tools. To show θ^{reg} is decreasing in β^{reg}, we have:

$$\frac{\partial \theta}{\partial \beta} = -\frac{\mathcal{D} - X_S^{eff}}{X_S^{eff}} (1-\beta)^{-2}$$

which is negative since $\mathcal{D} \geq X_S^{eff}$ by the assumption $\mathcal{E} \leq X_F^{eff}$.

With regard to the deposit insurance payment when the representative bank's pool of loan fails, we have $\frac{\partial \pi^H}{\partial \beta} = C\beta^{-2}$, which is positive since $C = \frac{\omega_L E[R^{eff}] X_S^{eff}}{q\mathcal{D}} > 0$. To show that the total lump-sum taxation in the high state is increasing in β^{reg}, it is enough to show that $R_D^{s,reg}$ is increasing in β^{reg}, which is equivalent to showing that $(\frac{\omega_H E[\widetilde{R}^{eff}]}{q\mathcal{D}} - \frac{E[\widetilde{R}^{eff}]}{p\mathcal{D}}) < 0$. The inequality holds since $p\omega_H < p\omega_H + (1-p)\omega_L = q$. The last two statements of the corollary follows from these relations and the fact that tje expected payout to deposit holders does not vary with β^{reg}.

Proof of Theorem 5

As $\tau > \bar{\tau}$ increases, by Proposition 2, we know that the investment in the safe asset increases, which means both $\frac{X_S^{eff}}{\mathcal{D}+\mathcal{E}}$ and $\frac{X_S^{eff}}{\mathcal{D}}$ are increasing. Hence, the interval for possible values of β^{reg} shifts to right. Moreover, for fixed β^{reg}, the corresponding capital requirement θ^{reg} can be written as $c_1 - c_2 \frac{1}{X_S^{eff}}$, where $c_1 = 1 + \frac{\beta^{reg}}{1-\beta^{reg}}$ and $c_2 = \frac{\beta^{reg}\mathcal{D}}{1-\beta^{reg}} > 0$. As τ increases, X_S^{eff} increases, which implies that θ^{reg} increases. Hence, the capital requirement becomes tighter.

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