Feedback Effect and Investor Information Acquisition: Implications for Agency Problems*

Jesse Davis[†] and Naveen Gondhi[‡]

March 15, 2018

Abstract

Despite the empirical prevalence of debt overhang, existing research has found little evidence of risk-shifting. To understand this discrepancy, we augment a traditional feedback model with an important feature: investors' endogenous learning. We show that more ex-ante inefficient opportunities for risk-shifting encourage information acquisition. This lowers the ex-post likelihood a firm's manager will choose such inefficient investments, attenuating risk-shifting. With debt overhang, this flips: more efficient projects discourage information acquisition. This increases the likelihood the manager forgoes efficient investment, amplifying debt overhang. Our analysis suggests a novel channel through which financial markets can differentially affect agency frictions between firm stakeholders.

JEL Classification: D82, G14, G32

Keywords: Feedback effect, Information acquisition, Agency problems, Debt Overhang, Risk-shifting, Information sensitivity

*We would like to thank Snehal Banerjee, Philip Bond and seminar participants at INSEAD, University of Lugano and Collegio Carlo Albero for valuable feedback.

[†]University of North Carolina - Chapel Hill (Jesse_Davis@kenan-flagler.unc.edu)

[‡]INSEAD (naveen.gondhi@insead.edu)

1 Introduction

There is now a growing literature which argues that financial markets, and the private information of investors contained in secondary market prices, can serve as a valuable source of information for firm managers.[1] While it is well-known that investors' incentive to acquire such information generally increases with the volatility of cash flows, this "feedback effect" literature suggests that such cash flow characteristics are endogenous, dependent upon the information acquired.[2,3] This creates a feedback loop. In the presence of risky debt, investments that increase the volatility of cash flows also increase the value of equity, suggesting that such feedback would be valuable to managers and investors alike. On the other hand, as the theory of risk-shifting (Jensen and Meckling (1976)) highlights, such investment projects may also be socially inefficient. Thus, understanding how investors' endogenous learning affects the likelihood of an agency conflict between the firm's stakeholders is both a natural and important issue for study.

In this paper, we develop, and establish the existence of, a non-linear rational expectations equilibrium which incorporates a feedback loop between the price of equity and the firm's investment decision.[4] In the presence of risky debt, learning from prices generically eliminates some inefficient investment decisions. Investors' endogenous learning, however, plays a crucial role in determining the extent to which this feedback arises across different types of investments. In particular, we show that while the *most inefficient risk-shifting projects are least likely to be adopted* after observing prices, the opposite is true *when debt overhang is feasible: the most efficient investments are most likely to be abandoned*. Consistent with these predictions, the empirical literature has thus far found evidence consistent with debt overhang (e.g., Moyen (2007)) but little support for risk-shifting (e.g., Gilje (2016)).[5] Our paper provides a single, novel channel through which such a disparity arises.

[1] See Bond, Edmans, and Goldstein (2012) for a survey of this literature.

[2] There is a large literature consistent with this general observation, starting with Grossman and Stiglitz (1980), Hellwig (1980) and corroborated by more recent work including Kacperczyk, Van Nieuwerburgh, and Veldkamp (2016)

[3] One notable example of the latter is Dow, Goldstein, and Guembel (2017), discussed in detail below.

[4] Bond, Goldstein, and Prescott (2009) shows that when prices reflect both firm fundamentals and future investment decisions, there may not be an equilibrium, since the price can exhibit non-monotonicity. We provide conditions under which this problem can be circumvented.

[5] The following section provides a more detailed exploration of this literature.

We consider a three-date (two-period) model. At date zero, the firm owns an existing asset and has access to a potential investment. While the firm manager and investors share common prior beliefs about the investment, each (competitive) investor can acquire costly, private information about the project's likelihood of success. At date zero, each investor chooses how much information to acquire in anticipation of trading an equity claim in the next period. At date one, the firm manager must decide whether or not to invest in the new project, and can use the information contained in the price of equity when doing so.[6] Investors incorporate this feedback loop into their demand schedules and the manager's decision is reflected in the price. At date two, the cash flows of any assets owned by the firm are realized and the proceeds are paid to existing debt and equity investors.

The extent to which the investment decision depends upon prices depends upon the quality of the information contained therein. As investors acquire more information, the manager conditions more heavily on the price. Note, though, that investors only want to invest in private information when the value of the traded claim is sensitive to the signal they receive. Importantly, we consider investment projects which can amplify or attenuate the information sensitivity of equity, depending upon the investment's payoff distribution. This proves to be the crucial distinction between risk-shifting and debt overhang in our setting. Consistent with our intuition regarding the volatility of cash flows, projects subject to risk-shifting increase the information sensitivity of equity, while investments subject to debt overhang cause it to decrease. This leads to ex-ante endogenous variation in investors' private information which, in turn, generates ex-post variation in the likelihood that the manager makes the investment.

We begin our main analysis by showing that, all else equal, the most inefficient forms of risk-shifting, i.e., those projects with the most negative ex-ante net present value, are least likely to be chosen inefficiently *after the manager conditions on prices.* A risk-shifting project transfers cash flows from bad to good states of the world, which increases the information sensitivity of equity. Moreover, the more ex-ante inefficient the project, the larger this change in information sensitivity. This increases the marginal value of acquiring information for equity holders, leading to more infor-

[6] We assume the manager makes investment decisions which maximize the expected value of equity, i.e., no agency conflict exists between firm managers and equity holders.

mative prices. As a result, the firm manager conditions more heavily on the price, which increases the variance of his posterior beliefs.[7] We show that as the variance of the manager's beliefs grows, investments that meet the manager's break even threshold are also more likely to be ex-post efficient. Thus, more inefficient projects have a higher likelihood of being crowded out by the information contained in prices.

On the other hand, the manager is most likely to forgo the most efficient investments when they are subject to debt overhang. The argument closely follows the logic above. Conditional on investment, a project which exhibits the potential for debt overhang decreases the information sensitivity of equity. The more efficient the project is ex-ante, the larger the fall in both information sensitivity and investor information acquisition. As a result, even after conditioning on prices, the manager is more likely to inefficiently opt out of investment: lower-quality information implies that the manager is more likely to stick with his ex-ante decision. In short, this suggests that endogenous information acquisition increases the likelihood that the worst examples of debt overhang persist.

Our model suggests that this difference in the prevalence of risk-shifting and debt overhang is more likely to arise when the firm has publicly-, not privately-held equity. Further, our results will be more pronounced in settings where investors have access to payoff-relevant information that managers do not possess. For instance, Luo (2005) provides evidence that an acquisition is more likely to be canceled if the market reacts negatively, particularly in cases where learning is more probable.

Finally, we note two additional contributions of our model to the theoretical literature. First, our model generalizes the analysis of Dow et al. (2017), which shows that markets with feedback can generate complementarity in investor information acquisition. In addition to the standard strategic substitutability, such as that found in Grossman and Stiglitz (1980), the ex-ante likelihood of investment success can generate strategic complementarity. When the ex-ante fundamentals of a project are weak, the firm only invests if the information in prices suggests that it is profitable to do so; as a result, Dow et al. (2017) show that the marginal value of learning about the project can increase when other investors produce information, as this increases the chance that the firm will make the

[7] If the manager could not condition on prices, he would invest in these ex-ante inefficient projects with certainty: doing so increases the expected value of equity.

investment. Hence, strategic complementarity arises across investors. Our model generalizes this result but provides an important counterpoint. By incorporating existing assets, we are able to show that this result depends upon the sign of the correlation between the return of the investment and the cash flows generated by assets-in-place. When the investment return is negatively correlated with that of assets-in-place, strategic complementarity can only arise with ex-ante stronger, not weaker, fundamentals.

Second, solving the model required the development of a new non-linear rational expectations equilibrium. The REE literature, including those models with a feedback effect, has long relied on an elegant linear framework. This framework, however, struggles to accommodate agency conflicts, which rely on the existence of risky debt and equity: non-linear claims. To confront this challenge, the first part of the paper extends the model of Davis (2017) to create a novel, tractable, non-linear REE with debt, equity and a feedback loop between security prices and the firm's investment decision. This model has the potential to answer a number of important research questions in which the presence of risky debt is an essential ingredient.

1.1 Related Literature

At its core, our model emphasizes the role played by financial markets in aggregating and disseminating information, following Grossman (1976), Grossman and Stiglitz (1980), Hellwig (1980) and Diamond and Verrecchia (1981). More recently, a theoretical literature has emerged which studies the role of secondary financial markets as an important source of information for decision makers, including firm managers (as in our model). Bond et al. (2012) provide a comprehensive survey of the "feedback effect" literature: below, we highlight those papers which most closely resemble our own.

As in Bond et al. (2009), Goldstein, Ozdenoren, and Yuan (2013), Bond and Goldstein (2015) and Dow et al. (2017)), investors in our model act competitively; the private information they possess is impounded into the price through their trading activity in a non-strategic manner.[8] Similar to the analysis of Bond et al. (2009), we show that our rational expectations pricing function has the

[8] In contrast, investors have price impact and act strategically in Goldstein and Guembel (2008), Edmans, Goldstein, and Jiang (2015) and Boleslavsky, Kelly, and Taylor (2017)).

potential to exhibit non-montonicity: the existence, therefore, of a feedback equilibrium requires some restrictions on the project characteristics. We show that, like Goldstein et al. (2013) and Dow et al. (2017), the feedback effect has the potential to create strategic complementarities across investors. In Goldstein et al. (2013), this complementarity arises through trading behvaior, whereas in our model (and in Dow et al. (2017)), this arises through the information acquisition decision of investors. Unlike Goldstein et al. (2013), however we allow the firm to have existing assets, which we show is crucial in determining under what conditions (positive of negative NPV) complementarity arises.

Myers (1977) argued that, in the presence of risky debt, equity holders exhibit debt overhang when they forego positive NPV projects in which the gains generated by their new investment will largely accrue to the existing debt holders. On the other hand, the theory of risk-shifting (Jensen and Meckling (1976)) suggests that managers can increase the value of shareholders' equity by pursuing some negative NPV projects in which the losses generated will largely accrue to debt holders. We show that allowing firm managers to learn from prices can reduce both activities; however, accounting for endogenous information acquisition, we show that the most egregious cases of risk shifting are largely eliminated while the likelihood of debt overhang is amplified. The latter is consistent with the empirical literature, including Mello and Parsons (1992), Parrino and Weisbach (1999), and Moyen (2007), who find evidence of debt overhang, as well as Andrade and Kaplan (1998), Rauh (2008) and Gilje (2016), who find little evidence for risk-shifting.

The existing theoretical literature has suggested other possible explanations for why we may not observe risk-shifting. In dynamic settings, both Diamond (1989) and Hirshleifer and Thakor (1992) consider the impact of reputational concerns on investment decisions. Similarly, Almeida, Campello, and Weisbach (2011) suggests that firms may reduce risk today so that positive NPV projects can be funded in the future. In contrast, we study a static setting and emphasize the mitigating role that prices (instead of project outcomes) can play in reducing risk-shifting.

Finally, our model focuses on the conflict between bond holders and equity holders; as a result, and unlike standard financial market models, in which prices and cash flows are linear, our framework must allow for non-linear claims (i.e., debt and equity). As such, it is most closely related to Davis (2017), Albagli, Hellwig, and Tsyvinski (2011) and Chabakauri, Yuan, and Zachariadis (2016). In this paper, we extend the model of Davis (2017). While both papers emphasize the importance of endogenous investor information acquisition, the focus of Davis (2017) is the firm's optimal issuance policy (post-investment) while we examine the firm's investment decision. Moreover, our extension allows for feedback between the manger's investment decision and the price, a feature Davis (2017) does not consider.

The reminder of the article is organized as follows. In section 2, we introduce the model. Section 3 establishes the existence of a feedback equilibrium and analyzes the investors' incentive to acquire information. In section 4, we apply our framework to understand the (relative) prevalence of agency frictions. Section 5 extends our analysis and section 6 concludes. All proofs can be found in the Appendix.

2 The Model

2.1 Model Setup

There are three dates, $t \in \{0, 1, 2\}$, and two states of the world, $s \in \{L, H\}$. A firm owns a risky asset which generates a payoff, x; this asset represents the firm's assets in place. The distribution of this payoff is state-dependent: $x \sim G_H$ (in the high state) or $x \sim G_L$ (in the low state), where both G_s are known, non-degenerate distributions. It is without loss of generality to allow for limited liability: we assume $G_s(x) = 0$ for all $x < 0$. Agents in the model do not know $q \equiv \mathbb{P}[s = H]$ with certainty, but know that

$$q = \Phi[z] \quad z \sim \mathcal{N}(\mu_z, \tau_z^{-1})$$

where Φ is the CDF of a standard normal distribution.

The firm also has access to a risky, state-dependent investment project which requires the firm to commit to an investment of I_y at date one.[9] At date two, the investment generates a cash flow of y_s,

[9]The investment is made using the firm's existing cash and does not require equity holders to contribute additional

which for tractability, and without loss of generality, is drawn from a degenerate distribution. We assume that the total distribution of cash flows in the high state first-order stochastically dominates the total distribution of cash flows in the low state, with or without investment.[10] As a result, given an agent's information set, $\mathcal{F}$, the NPV of the project can be written:

$$NPV|\mathcal{F} = \mathbb{E}[q|\mathcal{F}](y_H - I_y) + (1 - \mathbb{E}[q|\mathcal{F}])(y_L - I_y)$$

If the required investment, I_y, is smaller (greater) than the payoff in either states, y_H and y_L, then it is always (never) optimal to invest, eliminating any potential feedback effect. This leaves two cases to consider.

Case 1 ($y_H > I_y > y_L$): In this case, investment increases the firm's value in the high state. This implies that the cash flows of the project are positively correlated with the cash flows generated by the assets in place. Such an investment could be viewed as an amplifying investment, or a"doubling down", on the firm's assets in place. Alternatively, it could be said that the degree of correlation here represents the extent to which the *information* about the existing asset's payoff is correlated with the investment. Under this assumption, investment is efficient if and only if

$$\mathbb{E}[q|\mathcal{F}] > \frac{I_y - y_L}{y_H - y_L} \tag{1}$$

Case 2 ($y_H < I_y < y_L$): In this case, investment increases the firm's value in the low state: there is now a negative correlation between the cash flows of the project and those generated by the existing asset. Such an investment could be viewed as a corrective action taken by the firm, similar to that decribed in Bond et al. (2009). In particular, the benefit of this corrective action is high when the firm's fundamentals are low. Under this assumption, investment is efficient if and only if

$$\mathbb{E}[q|\mathcal{F}] < \frac{I_y - y_L}{y_H - y_L} \tag{2}$$

capital. We assume that the payoff, x, includes these cash holdings.

[10]Specifically, we assume that $G_L(x - y_L) > G_H(x - y_H)$ and $G_L(x) > G_H(x)$.

In a first-best world, the firm would follow the decision rules above. We assume, however, that the investment decision is made by a risk-neutral manager who owns an equity stake in the firm. As a result, he makes his investment decision based upon its impact on the expected value of equity; he will not, in general, follow the first-best policy.[11] In what follows, we will consider settings in which the date zero NPV of the project is negative (which could potentially lead to risk-shifting) and positive (so that debt overhang is a possibility).

The manager takes as given the firm's capital structure: specifically, outstanding equity and any previously issued debt.[12] Without loss of generality, we assume this outstanding liability is zero-coupon debt with a face-value of F due at date two. Given an agent's information set, $\mathcal{F}$, we can express the expected value of equity as

$$V(Equity|\mathcal{F}) = \begin{cases} \mathbb{E}[q|\mathcal{F}]E_H(F,0) + (1-\mathbb{E}[q|\mathcal{F}])E_L(F,0) & \text{absent investment} \\ \mathbb{E}[q|\mathcal{F}]E_H(F,y_H - I_y) + (1-\mathbb{E}[q|\mathcal{F}])E_L(F,y_L - I_y) & \text{with investment} \end{cases}$$

where $E_s(F,c) = \int_{F-c}^{\infty}(x - F + c)dG_s$.

In addition to the manager, there exists a unit-measure continuum of risk-neutral investors who, at date zero, share with the manager common prior beliefs about the likelihood of each state. Each investor, however, also has access to a private signal about the payoff's expected value. This information provides an incentive for the manager to learn from the price when making his investment decision. Specifically, investor $i \in [0,1]$ observes

$$s_i = z + \varepsilon_i \quad \varepsilon_i \sim \mathcal{N}(0, \tau_i^{-1})$$

By observing a private signal which conditions on z, investors can more precisely estimate from which distribution the firm's value will be drawn. Each investor can choose the precision of his signal ($\tau_i > 0$), but his choice is subject to a cost function, $C(\tau_i)$. We assume only that the cost function

[11]The manager's decision rule is specified in equations (4) and (5).

[12]This debt may have been previously issued to finance the existing cash flow. In future work, we hope to extend our model to allow the firm to choose an optimal capital structure, anticipating the feedback effect we analyze.

possesses standard characteristics: C is continuous, $C(0) = C'(0) = 0$, and $C', C'' > 0$ for all τ_i. The cost function is identical across investors.

2.2 Financial Market Equilibrium absent Feedback effect

For intuition, we begin by shutting down the feedback effect, i.e., firm managers choose their action without conditioning on the information contained in prices. We assume that investors can trade equity at date one.[13] They are subject to position limits; specifically, they can buy no more than one share and cannot short.[14] We will establish the existence of a non-linear rational expectations equilibrium in which each investor's demand conditions upon both his private signal as well as the information contained in the price, p_E. That is, for investor i, given an investment decision, the value of equity can be expressed:

$$E_L(F, c_L) + \mathbb{E}[q|s_i, p_E]\Delta E(F, \mathbf{c}) \quad \text{where} \quad \Delta E(F, \mathbf{c}) \equiv E_H(F, c_H) - E_L(F, c_L)$$

If the manager invests, $c_H = y_H - I_y, c_L = y_L - I_y$, and $\mathbf{c} = [c_L \; c_H]$; absent investment, all of these parameters are equal to zero.

Investors possess private information about the realization of q; moreover, it is easy to see that the sensitivity of each agent's valuation with respect to $\mathbb{E}[q|\mathcal{F}]$ is $\Delta E(F, \mathbf{c})$. As a result, we will refer to $\Delta E(F, \mathbf{c})$ as the **information-sensitivity of equity**.

It is straightforward to show the following:

Lemma 2.1. *(1) The information-sensitivity of equity is decreasing in the face value, F.*
(2) The information-sensitivity of equity is increasing in c_H and decreasing in c_L.

By analogy, we define the **information sensitivity of investment** as $y_H - y_L$.[15] This yields the following corollary:

[13] For now, we assume that the debt is held privately, for instance, by a bank. We hope to relax this assumption in future work.

[14] Both assumptions are without loss of generality in terms of our main comparative static: the impact of information sensitivity on information acquisition.

[15] The expected value of investing is $y_L + \mathbb{E}[q|\mathcal{F}]\,(y_H - y_L) - I_y$.

Corollary 2.2. *The information sensitivity of equity, given investment, is increasing in the information sensitivity of the project.*

In order to keep the price of equity from being fully revealing, we assume that there are also noise traders in the market who demand a fraction $\Phi(u)$ units of the outstanding equity; their demand is price-independent. We assume that $u \sim \mathcal{N}(0, \tau_n^{-1})$.

We will conjecture and verify that investors can construct a signal s_E from the price of equity, and that this signal will be normally-distributed and independent of s_i, conditional upon the true value, z. Let τ_E denote the precision of s_E, which is determined in equilibrium. Under this conjecture, each investor believes:

$$z|s_i, s_E \sim \mathcal{N}\left(\frac{\tau_z \mu_z + \tau_i s_i + \tau_E s_E}{\tau_z + \tau_i + \tau_E}, \frac{1}{\tau_z + \tau_i + \tau_E}\right)$$

As in Davis (2017), given an information set $\mathcal{F}$, it can be shown that

$$\mathbb{E}[q|\mathcal{F}] = \mathbb{E}[\Phi(z)|\mathcal{F}] = \Phi\left(\frac{\mathbb{E}[z|\mathcal{F}]}{\sqrt{1 + \mathbb{V}[z|\mathcal{F}]}}\right)$$

This implies that, given an investment decision, the expected value of equity, for investor i, can be written:

$$E_L(F, c_L) + \Phi\left(\frac{\tau_z \mu_z + \tau_i s_i + \tau_E s_E}{\sqrt{\psi\,(1+\psi)}}\right)\Delta E(F, \mathbf{c}) \quad \text{where} \quad \psi \equiv \tau_z + \tau_i + \tau_E$$

First-order stochastic dominance implies that $\Delta E(F, \mathbf{c}) > 0$, which in turn implies that the value of equity is increasing in the investor's conditional expectation of z. Investor beliefs can be ordered by their private signals and so we posit a threshold strategy: an investor purchases one unit of equity if $s_i \geq x(z, u)$; otherwise, they hold only the risk-free security (with return normalized to one). Note that the threshold is a function of both fundamentals (z) as well as the realized liquidity shock (u).

We normalize the outstanding supply of equity to one and impose market-clearing:

$$1 = \underbrace{\left[1 - \Phi\left(\sqrt{\tau_i}\,(x(z,u) - z)\right)\right]}_{\text{total demand by investors}} + \underbrace{\Phi(u)}_{\text{liquidity demand}}$$

Rewriting this expression shows that markets clear if and only if $x(z, u) = z + \frac{u}{\sqrt{\tau_i}}$. Moreover, the marginal investor, whose signal $s_i = x(z,u)$, sets the price equal to his conditional expectation given the investment decision:

$$p_E = E_L(F, c_L) + \mathbb{E}[q|s_i = x(z,u), p_E]\Delta E(F, \mathbf{c})$$

It is clear, therefore, that $x(z,u)$ is recoverable from the price, and so we write $s_E \equiv x(z,u)$. Moreover, s_E is normally-distributed, with precision $\tau_E = \tau_i \tau_n$ and mean z. This verifies our conjecture. Thus, we rewrite the price of equity:

$$p_E = E_L(F, c_L) + q_E \Delta E(F, \mathbf{c}) \quad \text{where} \quad q_E \equiv \Phi\left(\frac{\tau_z \mu_z + (\tau_i + \tau_E) s_E}{\sqrt{\psi\,(1+\psi)}}\right)$$

The formal definition of equilibrium without feedback effect is then as follows:

Definition 1. *A Perfect Bayesian Equilibrium for financial markets (given the investment decision of firm manager) consists of demand functions $d(s_i, p_E)$ for investors, a price function $p_E(z,u)$, and posterior beliefs such that (i) $d(s_i, p_E)$ is optimal given posterior beliefs; (ii) the asset market clears for all (z,u); and (iii) posterior beliefs satisfies Bayes' rule whenever applicable*

3 Feedback Effect Equilibrium

If the manager is able to condition on the price prior to making his investment decision, then a feedback loop is generated. Specifically, the information sensitivity of the security is a function of the manager's investment decision, which depends upon the information contained in the price. Of course, the manager's decision depends upon the quality of the signal contained in the price (i.e., investors' endogenous information acquisition), which is, in turn, a function of the information sensitivity. Hence, a feedback loop. We examine both aspects in this section.

3.1 Financial Market Equilibrium

We begin by analyzing the manager's investment decision, taking the investors' information acqusition decision as given.

In case 1, $y_H > I_y > y_L$: as a result, the value of equity increases in the high state and decreases in the low state, i.e.,

$$E_H(F, y_H - I_y) - E_H(F, 0) > 0 > E_L(F, y_L - I_y) - E_L(F, 0) \tag{3}$$

and the manager invests if and only if the high state is sufficiently likely. Specifically, when $y_H > I_y > y_L$, the manager invests if

$$\mathbb{E}[q|\mathcal{F}_m] > \frac{E_L(F, 0) - E_L(F, y_L - I_y)}{\Delta E(F, \mathbf{c}) - \Delta E(F, \mathbf{0})} \qquad \textbf{(Case 1)} \tag{4}$$

where $\mathcal{F}_m$ denote the manager's information set. Note that we can rewrite (3) to show that this condition also implies that the information sensitivity of equity increases with investment.

On the other hand, a case 2 investment (in which $y_L > I_y > y_H$) yields positive returns in the "low" state only. As a result, the manager must be sufficiently pessimistic about the likelihood of the "high" state to invest. This flips the inequality in (3) and reverses the cutoff for investment:

$$\mathbb{E}[q|\mathcal{F}_m] < \frac{E_L(F, 0) - E_L(F, y_L - I_y)}{\Delta E(F, \mathbf{c}) - \Delta E(F, \mathbf{0})} \qquad \textbf{(Case 2)} \tag{5}$$

Finally, we note that the manager's threshold belief (4 and 5) is common knowledge amongst all agents in the economy.

Investors have, and trade on, private information about the probability of each state. We conjecture that the manager can extract a signal $s_E \sim \mathcal{N}\left(z, \tau_E^{-1}\right)$ from the price. Under this assumption, managers' belief about the likelihood of the high state, given observation of s_E, can be written

$$\mathbb{E}[q|s_E] = \Phi\left(\frac{\tau_z\mu_z + \tau_E s_E}{\sqrt{(\tau_z+\tau_E)(1+\tau_z+\tau_E)}}\right) \tag{6}$$

We define $NPV_E = V(Equity|\text{with investment}) - V(Equity|\text{no investment})$, where the conditioning information is the shared prior beliefs of all agents. It is straightforward to derive the following result.

Lemma 3.1. *(1) Learning from prices* **decreases the likelihood of investment** *if* $NPV_E > 0$. *(2) Learning from prices* **increases the likelihood of investment** *if* $NPV_E < 0$.

If the manager is able to condition his investment decision on the price of equity as conjectured, the lemma above simply states that there will be a feedback effect. For instance, in case 1, if the information contained in prices is sufficiently pessimistic, then the manager chooses not to invest in the new project, even though his ex-ante beliefs indicated that it would be profitable to do so.[16]

If investors are aware of the relationship between prices and investment, they must account for it when determining their demand schedules. Specifically, in case 1, investors know that the manager will only invest if $\mathbb{E}[q|s_E] > K$, where we define

$$K \equiv \frac{E_L(F,0) - E_L(F, y_L - I_y)}{\Delta E(F, \mathbf{c}) - \Delta E(F, \mathbf{0})}$$

This is equivalent to the statement above, i.e., the manager only invests if the signal he obtains from the price is sufficiently optimistic; specifically, if

$$s_E > \frac{\Phi^{-1}(K)\left[\sqrt{(\tau_z+\tau_E)(1+\tau_z+\tau_E)}\right] - \tau_z\mu_z}{\tau_E} \equiv f(K, \tau_E) \tag{7}$$

We conjecture that investors are able to condition on the same information as the manager - they, too, can extract s_E from the price. As a result, they *know with certainty above what price the manager will choose to invest* when they submit their demand schedules. In particular, investment

[16] In case 2, the manager chooses to invest when the aggregated information is sufficiently pessimistic.

will only occur in case 1 if the belief of the marginal investor is sufficiently optimistic, i.e., if

$$q_E > \underline{q_E} \equiv \Phi\left(\frac{\tau_z \mu_z + (\tau_i + \tau_E) f(K, \tau_E)}{\sqrt{\psi\,(1+\psi)}}\right) \tag{8}$$

Following the same line of reasoning, in case 2, investment occurs when $q_E < \underline{q_E}$.

Note that each investors' conditional valuation of the traded equity remains monotonic in their belief about the true value of q. In case 1, as $\mathbb{E}[q|s_i, p_E]$ increases, the expected value of the assets in place increases; moreover, if p_E is sufficiently high (equivalently, if s_E is sufficiently high), the manager invests, further increasing both the expected value of equity as well as the information sensitivity. In case 2, as $\mathbb{E}[q|s_i, p_E]$ decreases, the expected value of the assets in place decreases; however, when p_E is sufficiently low (equivalently, if s_E is sufficiently low), the manager invests, which increases the expected value of equity, *relative to the value absent investment*. While this necessarily lowers the information sensitivity of equity, each investor's conditional value is still increasing in $\mathbb{E}[q|s_i, p_E]$ as long as the assumption of FOSD holds.[17] Both effects are clear in Figure 1, which is detailed below.

As above, we posit a threshold strategy in which investor i purchases equity if and only if $s_i \geq x(z,u)$. Following the same steps, the price of equity, as before, is simply the marginal investor's conditional value, which now accounts for the feedback effect. In case 1, we write

$$p_E(z,u) = \begin{cases} E_L(F,0) + \mathbb{E}[q|s_i = x(z,u), p_E]\Delta E(F, \mathbf{0}) & \text{if } q_E \leq \underline{q_E} \\ E_L(F,c_L) + \mathbb{E}[q|s_i = x(z,u), p_E]\Delta E(F, \mathbf{c}) & \text{if } q_E > \underline{q_E} \end{cases} \tag{9}$$

While in case 2, the investment policy is flipped:

$$p_E(z,u) = \begin{cases} E_L(F,c_L) + \mathbb{E}[q|s_i = x(z,u), p_E]\Delta E(F, \mathbf{c}) & \text{if } q_E \leq \underline{q_E} \\ E_L(F,0) + \mathbb{E}[q|s_i = x(z,u), p_E]\Delta E(F, \mathbf{0}) & \text{if } q_E > \underline{q_E} \end{cases} \tag{10}$$

As we show in the proof of Proposition 3.2, $x(z,u)$ remains recoverable from the price, verifying our

[17] This assumption is necessary for a financial market equilibrium to exist. Without it, the price would be non-monotonic in the marginal investor's belief about q.

conjecture regarding the information contained in the price.

Definition 2. *A Perfect Bayesian Equilibrium with feedback consists of functions $d(s_i, p_E)$, $p_E(z, u)$, an optimal investment decision for firm managers such that (i) all conditions in 1 are satisfied and (ii) most importantly, firm managers decision to invest is optimal given information in prices.*

Proposition 3.2. *In case 1 and 2, an equilibrium exists and is unique when*

$$\mu_z < \frac{\Phi^{-1}(K)}{\tau_z \tau_i} \left[(\tau_i + \tau_E)\sqrt{(\tau_z + \tau_E)(1 + \tau_z + \tau_E)} - \tau_E \sqrt{\psi(1+\psi)} \right] \tag{11}$$

Condition 11 ensures that the price weakly increases at the cutoff point which is necessary for the price function to be monotonic and invertible.

Figure 1 plots the price function in both cases. In both panels, the price function is monotonic in the information content, but exhibits a discontinuity at the threshold belief. This discontinuity arises because the manager's information set differs from that of the marginal investor, similar to what arises in Bond et al. (2009). In the first panel (case 1), the price steepens above $\underline{q_E}$, i.e., it becomes more sensitive to investors' private information; on the contrary, in the second panel (case 2), the price function flattens for those values of q_E below $\underline{q_E}$. This change in information sensitivity is due to the manager's investment decision, as described above.

3.2 Endogenous Information Equilibrium

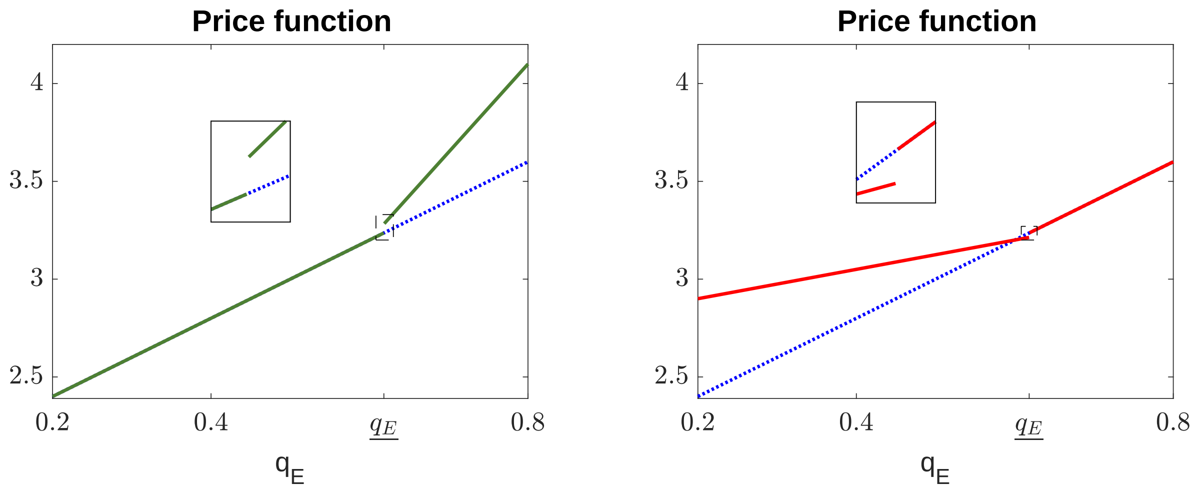
Given the financial market equilibrium established above, we can now analyze the investor's incentive to acquire information at date zero. The conditional expectation of an investor who observes private signal s_i, with precision τ_i, is given by

$$q_i = \mathbb{E}[\Phi(z)|s_i, s_E] = \Phi\left(\frac{\tau_z\mu_z + \tau_i s_i + \tau_E s_E}{\sqrt{\psi_i\left(1+\psi_i\right)}}\right)$$

where $\psi_i = \tau_z + \tau_i + \tau_E$. Recall that investors (i) differ only in their beliefs about z and (ii) purchase the asset only if their beliefs about z exceed those of the marginal investor. Then, in case 1, the

Figure 1: Price as a function of information content

The figure plots price as a function of information content for both cases. The solid line indicates the price path with the feedback effect. The dotted line indicates the hypothetical price absent the feedback effect, i.e., without investment. The relevant parameter values are $\tau_i = \tau_Z = \tau_n = 1$, $\mu_z = 0.2$, $E_L(F,0) = 2; E_H(F,0) = 4$. In case 1, $E_L(F,c) = 0.5; E_H(F,c) = 5$; in case 2, $E_L(F,c) = 2.75; E_H(F,c) = 3.5$.



(a) Case 1

(b) Case 2

investor's expected utility (expected trading gains) is

$$EU = \mathbb{E}[\Delta E(F,0)(q_i - q_E)\mathbb{1}_{q_i>q_E}\mathbb{1}_{q_E<\underline{q_E}} + \Delta E(F,c)(q_i - q_E)\mathbb{1}_{q_i>q_E}\mathbb{1}_{q_E>\underline{q_E}}] \tag{12}$$

$$= \mathbb{E}\left((q_i - q_E)\underbrace{\mathbb{1}_{q_i>q_E}}_{\substack{\text{buy if}\\ q_i>q_E}} \left[\Delta E(F,0)\overbrace{\mathbb{1}_{q_E<\underline{q_E}}}^{\substack{\text{Do not}\\ \text{Invest}}} + \Delta E(F,c)\overbrace{\mathbb{1}_{q_E>\underline{q_E}}}^{\text{Invest}} \right] \right) \tag{13}$$

where $F_x(y)$ is the cdf of random variable x evaluated at point y. The first term in equation 12 corresponds to the expected gain if the project is not taken; the second term is the expected gain given investment.

Proposition 3.3. *The marginal value of acquiring information is always positive.*

1. *In case 1, the marginal value of acquiring information increases with y_H and decreases with I_y.*

2. *In case 2, the marginal value of acquiring information decreases with y_L.*

Unsurprisingly, learning is valuable for investors. The proposition above also details how project characteristics affect the value of information. In general, we know that the more sensitive the security's price to information, the more valuable it is for an investor to acquire it. Consider the first case, in which investment increases information sensitivity. As y_H increases, both (i) the information sensitivity of equity (conditional on investment) and (ii) the likelihood of investment increase. As a result, the marginal value of acquiring information increases. On the other hand, an increase in I_y lowers both and so the marginal value falls. Finally, note that while an increase in y_L lowers the information sensitivity of equity (conditional on investment) it makes investment more likely, which leads to an ambiguous effect on the marginal value of information.

On the other hand, in case 2, investment decreases the information sensitivity of equity. Here, an increase in y_L increases the likelihood of investment and decreases the information sensitivity (conditional on investment) and so decreases the marginal value of information acquisition. For reasons echoing the logic of y_L in case 1, the impact of changes in y_H and I_y in case 2 are ambiguous.

We now establish the existence of an information acquisition equilibrium. Each investor chooses τ_i to maximize $EU(\tau_i, \tau_E) - C(\tau_i)$, taking all other investors choices as given. Specifically, let $\tau_E = \tau_{-i}\tau_n$, where τ_{-i} is the precision chosen by all other investors. We establish now the existence of a symmetric equilibrium in which all investors acquire signals of the same precision, i.e. $\tau_i = \tau_{-i}$.

Proposition 3.4. *There is a unique, symmetric equilibrium in information acquisition as long as* $\frac{\partial^2 EU}{\partial \tau_i \partial \tau_E} < 0$*, i.e., as long as information acquisition exhibits substitutability aross investors.*

In Section 4, we show that settings in which agency problems can arise necessarily demonstrate substitutability and therefore there is a unique equilibrium. However, in Section 5, we examine under what conditions complementarity can arise and consider its implications.

4 Agency Problems

We turn now to the main analysis of the paper: the effect of endogenous information acquisition, in combination with the feedback effect, on the likelihood of inefficient investment. In particular, we focus on two commonly-studied settings which arise in the presence of risky debt: risk-shifting, as in Jensen and Meckling (1976) and debt overhang, as in Myers (1977). We follow the conventions of the literature in defining both terms.

Definition 3. **Risk-shifting** *exists when an inefficient investment increases the value of equity* ($NPV_E > 0$ *and* $NPV < 0$)*, while* **debt overhang** *arises when an efficient investment lowers the value of equity* ($NPV_E < 0$ *and* $NPV > 0$)*.*

We begin by establishing under what assumptions these agency conflict can arise in our model.

Lemma 4.1. *In case 1, risk-shifting, but not debt overhang, is feasible. In case 2, debt overhang, but not risk-shifting, is feasible.*

When investment success is positively correlated with the value of existing assets, equity holders earn a larger share of the payoff contingent upon success but, in the presence of risky debt, absorb a lower share of the loss if the project fails. As a result, a project may be viewed favorably by equity holders while debt holders (or a social planner) may wish to stop such risk-shifting. On the other hand, when investment success is negatively correlated with the value of assets in place, the holders of risky debt may be able to claim a larger share of the payoff when the project succeeds, while absorbing a smaller share of the loss. As a result, a project which is viewed favorably by debt holders (or a social planner) may not be chosen by the manager, who holds equity, i.e. they may exhibit debt overhang.

The corollary to Lemma 4.1 captures how the feedback effect can reduce the agency conflict by providing more information to the manager.

Corollary 4.2. *(1) In case 1, any project which is "crowded out" (i.e.,* $\mathbb{E}[q|s_E] < \overline{q_E} < \mathbb{E}[q]$*) is inefficient (i.e.,* $NPV|s_E < 0$*).*
(2) In case 2, any project which is "crowded in" (i.e., $\mathbb{E}[q|s_E] < \underline{q_E} < \mathbb{E}[q]$*) is efficient (i.e.,* $NPV|s_E > 0$*).*

In essence, allowing the manager to condition on prices encourages more efficient investment decisions. In particular, allowing the manager to condition on the price (in case 1) can eliminate some cases of risk-shifting by providing information which discourages the manger from making the investment. Similarly, the feedback effect can eliminate some examples of debt overhang (in case 2) by providing sufficiently positive information such that the manager chooses to invest. As the previous section emphasizes, the extent to which the manager conditions on the price depends upon the quality of the information found therein which, in turn, depends upon the project characteristics. In what follows, we explore how one particular project characteristic (the ex-ante NPV, or the "efficiency" of the project) alters information acquisition and therefore attenuates (or amplifies) the agency problem under study.

Finally, before moving forward we provide the following proposition which ensures that there is a unique information acquisition equilibrium in the settings we choose to analyze.[18]

Proposition 4.3. *Information acquisition exhibits strategic substitutability in (i) case 1 when* $NPV_E > 0$ *and (ii) case 2 when* $NPV_E < 0$.

4.1 Risk-shifting

The proof of Lemma 4.1 shows that, in case 1, while risk-shifting is feasible, $NPV_E < 0 \implies NPV < 0$: any time the manager chooses not to invest it is efficient to abstain. In determining the severity of the agency problem, therefore, we want a measure which captures how often a chosen investment is inefficient.

We propose two such metrics. The first is the ex-ante likelihood that the investment undertaken by the firm manager is inefficient:

$$\mathbb{P}(\text{Inefficient Investment}) = \mathbb{P}\left(E(NPV_E|s_E) > 0 \text{ and } E(NPV|s_E) < 0\right) \tag{14}$$

$$= \mathbb{P}\left(E(q|s_E) > K \text{ and } E(q|s_E) < \frac{I_y - y_L}{y_H - y_L} \equiv K_0\right) \tag{15}$$

Note that K_0 is the efficient threshold proposed in Section 2. It is easy to see, from the proof of Lemma 4.1 that $K < K_0$ in case 1. As a result, this probability is always positive: some risk-shifting

[18]Note that risk shifting requires $NPV_E > 0$ while debt overhang requires $NPV_E < 0$.

will always exist. Using a change of variables, we can rewrite the probability of inefficient investment as

$$\int_{f(K,\tau_E)}^{f(K_0,\tau_E)} dF_{s_E} = \Phi\left(\frac{f(K_0,\tau_E)-\mu_z}{\sqrt{\tau_z^{-1}+\tau_E^{-1}}}\right) - \Phi\left(\frac{f(K,\tau_E)-\mu_z}{\sqrt{\tau_z^{-1}+\tau_E^{-1}}}\right) \tag{16}$$

where dF_{s_E} is the cdf of distribution of s_E.

The second metric is the conditional probability that an investment is inefficient given that an investment was made:

$$\mathbb{P}(\text{Inefficient Investment}|\text{Investment} > 0) = \frac{\mathbb{P}\left(K_0 > \mathbb{E}\left[q|s_E\right] > K\right)}{\mathbb{P}(\mathbb{E}\left[q|s_E\right] > K)} \tag{17}$$

$$= \frac{\Phi\left(\frac{f(K_0,\tau_E)-\mu_z}{\sqrt{\tau_z^{-1}+\tau_E^{-1}}}\right) - \Phi\left(\frac{f(K,\tau_E)-\mu_z}{\sqrt{\tau_z^{-1}+\tau_E^{-1}}}\right)}{1-\Phi\left(\frac{f(K,\tau_E)-\mu_z}{\sqrt{\tau_z^{-1}+\tau_E^{-1}}}\right)} \tag{18}$$

Importantly, note that both measures utilize the ex-post information set of the firm manager, i.e., after he observes the price of equity.[19]

In what follows, we aim to understand how the feedback effect affects projects of different quality. In particular, we analyze whether endogenous learning amplifies or attenuates the benefit of the feedback effect. Because the NPV of the project is a function of many variables, we parameterize our model in such a way that a single variable will serve as a proxy for the investment's efficiency.[20,21]

First, we define θ such that $E_L(F, c_L) - E_L(F, 0) = \theta_L(c_L)$ while $E_H(F, c_H) - E_H(F, 0) = \theta_H(c_H)$.

[19]Since an econometrician can observe prices, and therefore infer the manager's beliefs, the measure utilized in the model should also incorporate this conditioning information.

[20]Taking the partial derivative with respect to a function of many variables is not a well-defined object.

[21]In what follows, we fix the investment threshold K and change the NPV of the project. In an online appendix, we fix the NPV_E and alter the NPV.

Second, let $q_0 \equiv \Phi\left(\mu_z\sqrt{\frac{\tau_z}{1+\tau_z}}\right)$. Then,

$$NPV_E(Project) = q_0\theta_H + (1-q_0)\theta_L \tag{19}$$

$$NPV(Project) = q_0 c_H + (1-q_0)c_L \tag{20}$$

$$K = \frac{E_L(F,0) - E_L(F,c_L)}{\Delta E(F,[c_H,c_L]) - \Delta E(F,\mathbf{0})} = \frac{-\theta_L}{\theta_H - \theta_L} \tag{21}$$

For risk shifting to arise, we need $\theta_L < 0 < \theta_H$ (as well as $NPV_E > 0$ and $NPV < 0$). Given $\alpha > 0$, let

$$\theta_L = -\alpha \quad \text{and} \quad \theta_H = \alpha(1+\gamma).$$

As α increases, the investment effectively transfers cashflows from the low state to the high state. Moreover, as long as $q_0 > \frac{1}{\gamma+2}$, then $NPV_E = \alpha(2q_0 + q_0\gamma - 1)$ is positive and increasing with α. Finally, the lemma below establishes that α is also a proxy for increasingly inefficient investments.

Lemma 4.4. *The parameter α is a proxy for increasing inefficiency in the presence of risk-shifting opportunities, i.e. if the ex-ante $NPV_E > 0$ and ex-ante $NPV < 0$, then $\frac{\partial NPV}{\partial \alpha} < 0$.*

Intuitively, while equity holders benefit from successful outcomes of high-risk (high α) projects, the losses from unsuccessful outcomes are borne by debt holders. Furthermore, not only is there a transfer of wealth from debt holders to equity holders but there is a reduction in enterprise value - as α increases, these projects becomes increasingly more socially inefficient.

Investors also account for the change in α when they decide how much information to acquire. First, as α increases, it is straightforward to see that the information sensitivity of the project increases. Moreover, the likelihood of investment also increases. While in this parameterization we have fixed K, the threshold belief about the probability of the "high" state, the NPV_E of the project is actually increasing, which lowers the threshold *price* at which investment occurs.[22] Taken together, the following proposition tells us that, in the face of risk-shifting, investors acquire more information about less efficient projects.

[22] Specifically, as α increases, $f(K, \tau_E)$ falls. Note that in the presence of risk-shifting, this should bias against us finding our results.

Proposition 4.5. *Marginal value of acquiring information increases with α.*

Finally, we establish the last piece of our argument.

Proposition 4.6. *Inefficient investment decreases with more information acquisition:*
(1) the probability of inefficient investment falls as τ_E increases, and
(2) the conditional probability of inefficient investment falls as τ_E increases,
if $\mu_z \in [\underline{\mu}, \overline{\mu}]$, where $\underline{\mu}, \overline{\mu}$ are defined in the appendix.

For intuition, we consider what occurs when $\mu_z = 0$. In this case, the high and low state are ex-ante equally likely: in order for risk-shifting to arise it must be that $K < 0.5 < K_0$. Absent any information in prices, equity holders will surely invest: with probability one, the manager's belief lies above his investment threshold (K), but below the efficiency threshold (K_0). As the price becomes more informative, the manager conditions more heavily on the price, which increases the variance of his posterior beliefs; this, in turn, decreases the probability that the firm manager's posterior belief falls in the range $[K, K_0]$. As a result, both measures of inefficient investment will decrease as well.

In summary, as the risk shifting becomes more inefficient, investors choose to acquire more information. By acquiring more information, however, they make investment in such inefficient projects less likely. Finally, we note that the restrictions on μ_z in Proposition 4.6 arise due to the non-linear relationship between the information acquired and the expected payoff of the asset.[23]

4.2 Debt Overhang

Lemma 4.1 shows that in case 2, $NPV_E > 0 \implies NPV > 0$: any investment taken by the manager must be efficient. In determining the extent of the debt overhang problem, we choose a measure which captures how often an efficient investment is foregone.

[23] These restrictions ensure that the impact of the non-linearity, which manifests itself through Jensen's inequality as a change in the average conditional expectation, does not swamp the impact of learning, which increases the variation in conditional beliefs. Importantly, this is a restriction which arises due to a specific functional form and is not a restriction driven by the underlying economic mechanism.

We propose two such metrics. The first is the ex-ante likelihood that the investment not undertaken is efficient.

$$\mathbb{P}(\text{Efficient Investment not taken}) = \mathbb{P}\left(E(NPV_E|s_E) < 0 \text{ and } E(NPV|s_E) > 0\right) \tag{22}$$

$$= \mathbb{P}\left(E(q|s_E) < K \text{ and } E(q|s_E) > \frac{I_y - y_L}{y_H - y_L} \equiv K_0\right) \tag{23}$$

It is easy to see, from the proof of Lemma 4.1 that $K > K_0$ in case 2. As a result, this probability is always positive: some debt-overhang will always exist. We can rewrite the above probability as

$$\int_{f(K_0,\tau_E)}^{f(K,\tau_E)} dF_{s_E} = \Phi\left(\frac{f(K,\tau_E) - \mu_z}{\sqrt{\tau_z^{-1} + \tau_E^{-1}}}\right) - \Phi\left(\frac{f(K_0,\tau_E) - \mu_z}{\sqrt{\tau_z^{-1} + \tau_E^{-1}}}\right) \tag{24}$$

where dF_{s_E} is the cdf of distribution of s_E. The second metric is the conditional probability that an investment is efficient given that an investment was not made:

$$\mathbb{P}(\text{Efficient Investment not taken}|\text{Investment} = 0) = \frac{\mathbb{P}\left(K > \mathbb{E}\left[q|s_E\right] > K_0\right)}{\mathbb{P}(\mathbb{E}\left[q|s_E\right] < K)} \tag{25}$$

$$= \frac{\Phi\left(\frac{f(K,\tau_E)-\mu_z}{\sqrt{\tau_z^{-1}+\tau_E^{-1}}}\right) - \Phi\left(\frac{f(K_0,\tau_E)-\mu_z}{\sqrt{\tau_z^{-1}+\tau_E^{-1}}}\right)}{\Phi\left(\frac{f(K,\tau_E)-\mu_z}{\sqrt{\tau_z^{-1}+\tau_E^{-1}}}\right)} \tag{26}$$

We define q_0 and θ as in the previous section. For debt overhang to arise, we need $\theta_L > 0 > \theta_H$ (as well as $NPV_E < 0$ and $NPV > 0$). Given $\alpha > 0$, let

$$\theta_L = \alpha \quad \text{and} \quad \theta_H = -\alpha(1+\gamma).$$

As α increases, the investment effectively transfers cashflows from the high state to the low state. Moreover, as long as $q_0 > \frac{1}{\gamma+2}$, then $NPV_E = -\alpha\left(2q_0 + q_0\gamma - 1\right)$ is negative and decreasing with α. Finally, the lemma below establishes that α is also a proxy for increasingly efficient investments.

Lemma 4.7. *The parameter α is a proxy for increasing efficiency in the presence of debt-overhang* *opportunities, i.e. if the ex-ante* $NPV_E < 0$ *and ex-ante* $NPV > 0$*, then* $\dfrac{\partial NPV}{\partial \alpha} > 0$.

Intuitively, while debt holders benefit from successful outcomes of high-risk (high α) projects, the losses from unsuccessful outcomes are borne by equity holders. Furthermore, not only is there a transfer of wealth from equity holders to debt holders but there is a increase in enterprise value - as α increases, these projects becomes increasingly more socially efficient.

Investors also account for the change in α when they decide how much information to acquire. First, as α increases, it is straightforward to see that the information sensitivity of the project decreases. Moreover, the likelihood of investment also decreases. Taken together, the following proposition tells us that, in the face of debt-overhang, investors acquire less information about more efficient projects.

Proposition 4.8. *Marginal value of acquiring information decreases with* α*.*

Finally, we turn to the last piece of our argument.

Proposition 4.9. *Efficient investment falls with less information acquisition:*
(1) the probability of efficient investment not taken increases as τ_E *decreases, and*
(2) the conditional probability of efficient investment not taken increases as τ_E *decreases, if* $\mu_z \in \left[\underline{\mu}, \overline{\mu}\right]$*, where* $\underline{\mu}, \overline{\mu}$ *are defined in the appendix.*

For intuition, we consider what occurs when $\mu_z = 0$. In this case, in order for debt-overhang to arise it must be that $K_0 < 0.5 < K$. Absent any information in prices, the firm manager would never invest. With the information contained in prices, firm managers invest with some probability; however, as the price becomes less informative, the manager relies on his prior belief more. As a result, the likelihood that efficient investments are foregone increases.

In summary, as projects which exhibit the potential for debt overhang become more efficient, investors choose to acquire less information. By acquiring less information, however, they make investment in such efficient projects less likely.

5 Information Complementarity

In our framework, settings in which agency problems can arise necessarily exhibit substituability in information acqusition across investors. While such a result is common in the larger market microstructure literature, it stands in contrast to the results of (Dow et al., 2017), who emphasize the possibility of mulitple equilibria and the presence of complementarity in feedback models. In what follows, we show how our model can replicate their results as a special case and extend their analysis to account for the role of a firm's existing assets.

We begin by establishing conditions under which complementarity can arise.

Proposition 5.1. *For the marginal value of acquiring information to increase in the precision of others' information, i.e.* $\frac{\partial^2 EU}{\partial \tau_i \partial \tau_E} > 0$*, it must be that*

1. *the project is ex-ante suboptimal, i.e.* $NPV_E < 0$*, in case 1, and*

2. *the project is ex-ante optimal, i.e.* $NPV_E > 0$ *in case 2.*

To understand these results, it is useful to isolate the two economic forces in our setting that determine how others' information acquisition affects the marginal value of learning. First, there is the standard substitutability effect (such as that found in Grossman and Stiglitz (1980)) which decreases the marginal value of acquiring information: the price becomes more informative and so there is less value in private learning. Second, there is a novel effect due to the endogenous investment decision. In particular, the degree to which managers condition on the information contained in the price depends upon its quality. The direction of this second effect depends on two factors: the ex-ante NPV_E and the correlation between the assets in place and the investment payoff.

If the risky project is ex-ante optimal, the default decision is to take the project. Conditioning on the price introduces the possibility that the firm will choose not to invest and moreover, the likelihood of investment decreases when more precise information is available. In case 1, when the investment is positively correlated with the assets in place, this reduces the expected information sensitivity of equity, lowering each investor's incentive to learn. As a result, there is strategic substitutability across investors. On the other hand, if the project is ex-ante suboptimal, the firm's default choice is to pass on the investment. As a result, conditioning on a price which is more informative *increases* the possibility of investment, since it lowers the threshold price at which the manager will chooose to invest. In case 1, this increases the expected information sensitivity, which increases the marginal value of information. As a result, when others learn more it can "crowd in" private information. When this latter effect dominates the traditional Grossman-Stiglitz effect, learning exhibits complementarity.

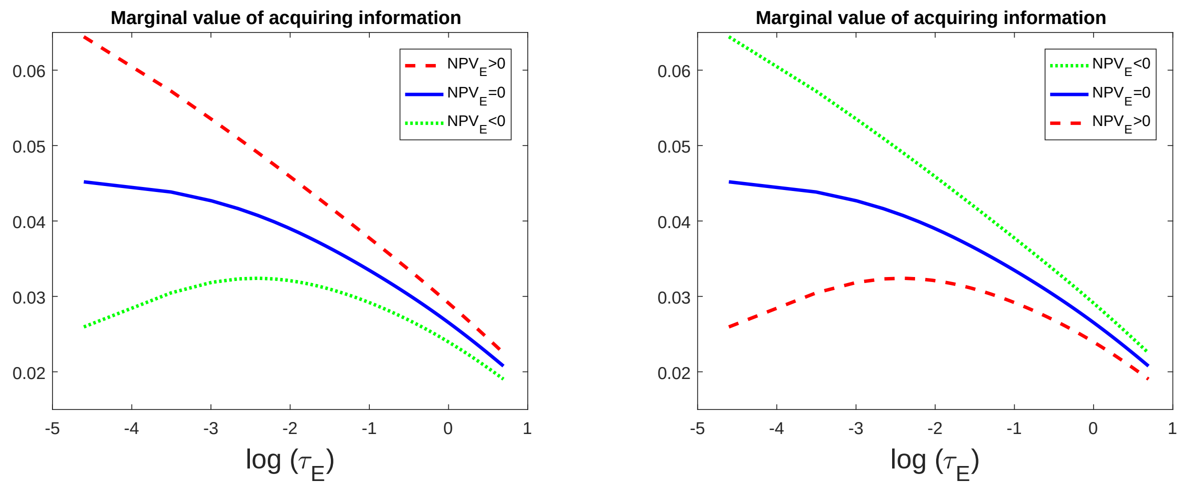
This result, and the possibility of multiple equilibria that it generates, is very similar to what is found in (Dow et al., 2017). In their setting, however, the firm does not have any assets in place; as a result, an investment project of *any* type increases the information sensitivity. Essentially, this corresponds to the first case in our model but sets $\Delta E(F, 0) = 0$, i.e., assets in place are informationally-insensitive.[24]

Our analysis generalizes their result but also extends the analysis to allow for investments which would *lower* information sensitivity. In particular, if the investment is negatively correlated with the firm's assets in place, as it is in the second case, the results of (Dow et al., 2017) are reversed. If the project is ex-ante suboptimal, as others learn more, investment becomes more likely, which *lowers* the expected information sensitivity. This discourages private information acquisition, in contrast to what arises in case 1. On the other hand, if the project is ex-ante optimal, more precise information in the price makes investment less likely, which *increases* the expected information sensitivity of equity. That is, in case 2, learning across investors exhibits strategic complementarity when the ex-ante NPV_E is positive.

Figure 2 provides a numerical illustration of these effects. In the first panel, investment is positively correlated with assets in place; as a result, the marginal value of learning increases with others' information acquisition only when the ex-ante NPV_E is sufficiently negative (the dotted line). Note that, eventually, as τ_E increases, the standard substitutability effect dominates so that the marginal value is non-monotonic in the information acquisition of others. In the second panel, where investment is negatively correlated with assets in place, this logic is reversed: complementarity only arises

[24] In our setting, $\Delta E(F, 0) = 0$ if the debt security operates as a pass-through.

Figure 2: Incentive to acquire information as a function of price informativeness
The figure plots the marginal value of acquiring information as a function of the precision of the information contained in the price for projects with differing levels of ex-ante profitability, i.e. NPV_E. Parameter values are $\tau_i = \tau_Z = \tau_n = 1$. In case 1, $\Delta E(F, c) = 3$, $\Delta E(F, c) = 1$ while in case 2, these are flipped.



(a) Case 1 (b) Case 2

when the NPV_E is sufficiently positive (the dashed line).

6 Conclusion

This paper argues that (i) when investors have access to endogenous information about investment opportunities and (ii) firm managers condition on such information when making investment decisions, risk-shifting should be mitigated while debt overhang can be worsened.

There are several promising directions for future research. First, we note that the manager's investment decision also affects the value of any debt claim, suggesting that allowing for traded debt may reveal information of interest to the firm. Second, we have assumed that the manager does not have access to any private information; as a result, observation of the price is sufficient to reveal whether or not investment will occur. We would like to explore how allowing the manager to acquire complementary information affects both investors' information acquisition problem as well as the existing agency conflicts.

References

E. Albagli, C. Hellwig, and A. Tsyvinski. A theory of asset prices based on heterogeneous information. NBER Working Paper No. 17548, 2011.

Heitor Almeida, Murillo Campello, and Michael S Weisbach. Corporate financial and investment policies when future financing is not frictionless. Journal of Corporate Finance, 17(3):675–693, 2011.

Gregor Andrade and Steven N Kaplan. How costly is financial (not economic) distress? evidence from highly leveraged transactions that became distressed. The Journal of Finance, 53(5):1443–1493, 1998.

Raphael Boleslavsky, David L Kelly, and Curtis R Taylor. Selloffs, bailouts, and feedback: Can asset markets inform policy? Journal of Economic Theory, 169:294–343, 2017.

Philip Bond and Itay Goldstein. Government intervention and information aggregation by prices. The Journal of Finance, 70(6):2777–2812, 2015.

Philip Bond, Itay Goldstein, and Edward Simpson Prescott. Market-based corrective actions. The Review of Financial Studies, 23(2):781–820, 2009.

Philip Bond, Alex Edmans, and Itay Goldstein. The real effects of financial markets. The Annual Review of Financial Economics is, 4:339–60, 2012.

Georgy Chabakauri, Kathy Yuan, and Konstantinos E Zachariadis. Multi-asset noisy rational expectations equilibrium with contingent claims. 2016.

Jesse Davis. Optimal issuance across markets and over time. Working Paper, 2017.

Douglas W Diamond. Reputation acquisition in debt markets. Journal of political Economy, 97(4): 828–862, 1989.

Douglas W Diamond and Robert E Verrecchia. Information aggregation in a noisy rational expectations economy. Journal of Financial Economics, 9(3):221–235, 1981.

James Dow, Itay Goldstein, and Alexander Guembel. Incentives for information production in markets where prices affect real investment. Journal of the European Economic Association, page jvw023, 2017.

Alex Edmans, Itay Goldstein, and Wei Jiang. Feedback effects, asymmetric trading, and the limits to arbitrage. The American Economic Review, 105(12):3766–3797, 2015.

Erik P Gilje. Do firms engage in risk-shifting? empirical evidence. The Review of Financial Studies, 29(11):2925–2954, 2016.

Itay Goldstein and Alexander Guembel. Manipulation and the allocational role of prices. The Review of Economic Studies, 75(1):133–164, 2008.

Itay Goldstein, Emre Ozdenoren, and Kathy Yuan. Trading frenzies and their impact on real investment. Journal of Financial Economics, 109(2):566–582, 2013.

Sanford Grossman. On the efficiency of competitive stock markets where trades have diverse information. The Journal of finance, 31(2):573–585, 1976.

S.J. Grossman and J.E. Stiglitz. On the impossibility of informationally efficient markets. The American Economic Review, 70(3):393–408, 1980.

Martin Hellwig. On the aggregation of information in competetive markets. Journal of Economic Theory, 3:477–498, 1980.

David Hirshleifer and Anjan V Thakor. Managerial conservatism, project choice, and debt. The Review of Financial Studies, 5(3):437–470, 1992.

Michael C Jensen and William H Meckling. Theory of the firm: Managerial behavior, agency costs and ownership structure. Journal of financial economics, 3(4):305–360, 1976.

Marcin Kacperczyk, Stijn Van Nieuwerburgh, and Laura Veldkamp. A rational theory of mutual funds' attention allocation. Econometrica, 84(2):571–626, 2016.

Yuanzhi Luo. Do insiders learn from outsiders? evidence from mergers and acquisitions. The Journal of Finance, 60(4):1951–1982, 2005.

Antonio S Mello and John E Parsons. Measuring the agency cost of debt. The Journal of Finance, 47(5):1887–1904, 1992.

Nathalie Moyen. How big is the debt overhang problem? Journal of Economic Dynamics and Control, 31(2):433–472, 2007.

Stewart C Myers. Determinants of corporate borrowing. Journal of financial economics, 5(2):147–175, 1977.

Robert Parrino and Michael S Weisbach. Measuring investment distortions arising from stockholder–bondholder conflicts. Journal of Financial Economics, 53(1):3–42, 1999.

Joshua D Rauh. Risk shifting versus risk management: Investment policy in corporate pension plans. The Review of Financial Studies, 22(7):2687–2733, 2008.

Appendix: Proofs

Proof of Lemma 2.1. The first statement is shown to be true in Davis(2017) as long as G_H first-order stochastically dominates G_L. To see the second statement, it is sufficient to show that $\frac{\partial E_s(F,c)}{\partial c} > 0$.

$$E_s(F,c) = \int_{F-c}^{\infty} (x - F + c) dG_s \implies$$
$$\frac{\partial E_s(F,c)}{\partial c} = [(F-c) - F + c] + \int_{F-c}^{\infty} 1 \, dG_s = 1 - G_s(F-c) > 0$$

■

Proof of Corollary 2.2. The information sensitivity of investment increases as y_H rises (which increases c_H) and as y_L falls (a decrease in c_L). By Lemma 2.1, we have our result. ■

Proof of Lemma 3.1. Suppose that $NPV_E > 0$. Then, absent prices, this project is always funded; however, there always exists a sufficiently low s_E such that $\mathbb{E}[q|s_E]$ no longer meets the required threshold. Furthermore, because s_E has full support on the real number line, this implies that the probability of investment is less than one. Similarly, when $NPV_E < 0$, the project is never funded absent prices, but there exists an s_E which is sufficiently high such that $\mathbb{E}[q|s_E]$ now meets the required threshold; as a result, the probability of investment is greater than zero. ■

Proof of Proposition 3.2. In case 1, the equilibrium exists when there is a price increase at $\underline{q_E}$ i.e., $E_L(F,0) + \mathbb{E}[q|s_i = x(z,u), p_E]\Delta E(F, \mathbf{0}) < E_L(F, c_L) + \mathbb{E}[q|s_i = x(z,u), p_E]\Delta E(F, \mathbf{c})$. This can be rewritten as

$$\mathbb{E}[q|s_i = x(z,u), p_E] > \mathbb{E}[q|p_E] = K$$
$$\iff \frac{\tau_z \mu_z + (\tau_i + \tau_E) f(K, \tau_E)}{\sqrt{\psi\,(1+\psi)}} > \Phi^{-1}(K)$$
$$\iff \tau_z \mu_z + (\tau_i + \tau_E) \frac{\Phi^{-1}(K)\left[\sqrt{(\tau_z + \tau_E)(1 + \tau_z + \tau_E)}\right] - \tau_z \mu_z}{\tau_E} > \Phi^{-1}(K)\sqrt{\psi\,(1+\psi)}$$

Simplifying this condition gives us 11. In case 2, the equilibrium exists when there is a price drop at

$\underline{q_E}$ i.e., $E_L(F,0) + \mathbb{E}[q|s_i = x(z,u), p_E]\Delta E(F,\mathbf{0}) > E_L(F,c_L) + \mathbb{E}[q|s_i = x(z,u), p_E]\Delta E(F,\mathbf{c})$. This can be rewritten as

$$\mathbb{E}[q|s_i = x(z,u), p_E] > \mathbb{E}[q|p_E] = K$$

Note that this is the same condition as in case 1 and simplifying this condition gives us 11. ■

Proof of Proposition 3.3. Expected utility in case 1 is given by

$$\begin{aligned} EU &= \mathbb{E}[\Delta E(F,0)(q_i - q_E)\mathbb{1}_{q_i>q_E}\mathbb{1}_{q_E<\underline{q_E}} + \Delta E(F,c)(q_i - q_E)\mathbb{1}_{q_i>q_E}\mathbb{1}_{q_E>\underline{q_E}}] \\ &= \Delta E(F,0)\int\limits_{-\infty}^{\underline{q_E}} dF_{q_E}(q_E)\int\limits_{q_E}^{\infty}(q_i - q_E)dF_{q_i|q_E}(q_i) + \Delta E(F,c)\int\limits_{\underline{q_E}}^{\infty} dF_{q_E}(q_E)\int\limits_{q_E}^{\infty}(q_i - q_E)dF_{q_i|q_E}(q_i) \end{aligned}$$

where $F_x(y)$ is the cdf of random variable x evaluated at point y. Note that

$$s_i|s_E \sim \mathcal{N}\left(\frac{\tau_z\mu_z + \tau_E s_E}{\tau_z + \tau_E}, \frac{1}{\tau_z + \tau_E} + \frac{1}{\tau_i}\right)$$

Let $w_i = \frac{\tau_z\mu_z + \tau_i s_i + \tau_E s_E}{\sqrt{\psi_i(1+\psi_i)}}$ and $w_E = \frac{\tau_z\mu_z + (\tau+\tau_E)s_E}{\sqrt{\psi(1+\psi)}}$. Then

$$w_i|s_E \sim \mathcal{N}\left(\sqrt{\frac{\psi_i}{1+\psi_i}}\frac{\tau_z\mu_z + \tau_E s_E}{\tau_z + \tau_E}, \frac{\tau_i}{(1+\psi_i)(\tau_z + \tau_E)}\right) \tag{27}$$

Expected utility can be rewritten as

$$EU(\tau_i) = \Delta E(F,0)\int\limits_{-\infty}^{f(\tau_E,K)} dF_{s_E}(s_E)\int\limits_{w_E}^{\infty}\{\Phi(w_i) - \Phi(w_E)\}dF_{w_i|s_E}(w_i) + \Delta E(F,c)\int\limits_{f(\tau_E,K)}^{\infty} dF_{s_E}(s_E)\int\limits_{w_E}^{\infty}\{\Phi(w_i) - \Phi(w_E)\}dF_{w_i|s_E}(w_i)$$

Define $H(s_E,\tau_E,\tau_i) = \int\limits_{w_E}^{\infty}\{\Phi(w_i) - \Phi(w_E)\}dF_{w_i|s_E}(w_i)$. Note that H is always positive. We can rewrite expected utility with this new notation as

$$EU(\tau_i,\tau_E) = \Delta E(F,0)\int\limits_{-\infty}^{f(\tau_E,K)} H(s_E,\tau_E,\tau_i)dF_{s_E}(s_E) + \Delta E(F,c)\int\limits_{f(\tau_E,K)}^{\infty} H(s_E,\tau_E,\tau_i)dF_{s_E}(s_E) \tag{28}$$

In case 2, the expected utility can be written as

$$
\begin{aligned}
EU &= \Delta E(F,c) \int_{-\infty}^{q_E} dF_{q_E}(q_E) \int_{q_E}^{\infty} (q_i - q_E) dF_{q_i|q_E}(q_i) + \Delta E(F,0) \int_{\underline{q_E}}^{\infty} dF_{q_E}(q_E) \int_{q_E}^{\infty} (q_i - q_E) dF_{q_i|q_E}(q_i) \\
&= \Delta E(F,c) \int_{-\infty}^{f(\tau_E,K)} H(s_E,\tau_E,\tau_i) dF_{s_E}(s_E) + \Delta E(F,0) \int_{f(\tau_E,K)}^{\infty} H(s_E,\tau_E,\tau_i) dF_{s_E}(s_E)
\end{aligned}
$$

1. Let the information set (filtration) $\mathcal{F}$ be more informative than $\mathcal{G}$ (i.e., $\mathcal{G}$ is a coarser filtration: $\mathcal{G} \subset \mathcal{F}$). Let a^F (and U^F) and a^G (and U^G) denote the optimal demands (and corresponding expected utilities) under filtrations $\mathcal{F}$ and $\mathcal{G}$. The fact that $\mathcal{G} \subset \mathcal{F}$ implies that $U^F \geq U^G$. Hence expected utility weakly increases with more information.

2. In case 1, taking partial derivative of expected utility with respect to y_H gives us

$$
\begin{aligned}
\frac{\partial EU}{\partial y_H} =& (\Delta E(F,0) - \Delta E(F,c)) H(f(\tau_E,K),\tau_E,\tau_i) f_{s_E}(f(\tau_E,K)) \frac{\partial f(\tau_E,K)}{\partial y_H} + \frac{\partial \Delta E(F,c)}{\partial y_H} \int_{f(\tau_E,K)}^{\infty} H(s_E,\tau_E,\tau_i) dF_{s_E}(s_E) \\
=& (\Delta E(F,0) - \Delta E(F,c)) H(f(\tau_E,K),\tau_E,\tau_i) f_{s_E}(f(\tau_E,K)) \frac{\sqrt{(\tau_z+\tau_E)(1+\tau_z+\tau_E)}}{\tau_E \phi(\Phi^{-1}(K))} \frac{\partial K}{\partial y_H} + (1 - G_H(F - y_H + I_y)) \int_{f(\tau_E,K)}^{\infty} H(s_E,\tau_E,\tau_i) dF_{s_E}(s_E) \\
=& \left\{ K H(f(\tau_E,K),\tau_E,\tau_i) f_{s_E}(f(\tau_E,K)) \frac{\sqrt{(\tau_z+\tau_E)(1+\tau_z+\tau_E)}}{\tau_E \phi(\Phi^{-1}(K))} + \int_{f(\tau_E,K)}^{\infty} H(s_E,\tau_E,\tau_i) dF(s_E) \right\} (1 - G_H(F - y_H + I_y)) \\
>& \; 0
\end{aligned}
$$

In case 1, taking partial derivative wrt I_y gives us

$$
\begin{aligned}
\frac{\partial EU}{\partial I_y} =& (\Delta E(F,0) - \Delta E(F,c)) H(f(\tau_E,K),\tau_E,\tau_i) f_{s_E}(f(\tau_E,K)) \frac{\partial f(\tau_E,K)}{\partial I_y} + \frac{\partial \Delta E(F,c)}{\partial I_y} \int_{f(\tau_E,K)}^{\infty} H(s_E,\tau_E,\tau_i) dF_{s_E}(s_E) \\
=& (\Delta E(F,0) - \Delta E(F,c)) H(f(\tau_E,K),\tau_E,\tau_i) f_{s_E}(f(\tau_E,K)) \frac{\sqrt{(\tau_z+\tau_E)(1+\tau_z+\tau_E)}}{\tau_E \phi(\Phi^{-1}(K))} \frac{\partial K}{\partial I_y} + (G_H(F - c_H) - G_L(F - c_L)) \int_{f(\tau_E,K)}^{\infty} H(s_E,\tau_E,\tau_i) dF_{s_E}(\\
<& \; 0
\end{aligned}
$$

In case 2, taking partial derivative of expected utility wrt y_L gives us

$$\begin{aligned}\frac{\partial EU}{\partial y_L} &= (\Delta E(F,c)-\Delta E(F,0))H(f(\tau_E,K),\tau_E,\tau_i)f_{s_E}(f(\tau_E,K))\frac{\partial f(\tau_E,K)}{\partial y_L}+\frac{\partial \Delta E(F,c)}{\partial y_L}\int_{-\infty}^{f(\tau_E,K)} H(s_E,\tau_E,\tau_i)dF_{s_E}(s_E)\\ &= (\Delta E(F,c)-\Delta E(F,0))H(f(\tau_E,K),\tau_E,\tau_i)f_{s_E}(f(\tau_E,K))\frac{\sqrt{(\tau_z+\tau_E)(1+\tau_z+\tau_E)}}{\tau_E\phi(\Phi^{-1}(K))}\frac{\partial K}{\partial y_L}-(1-G_L(F-y_L+I_y))\int_{f(\tau_E,K)}^{\infty} H(s_E,\tau_E,\tau_i)dF_{s_E}(s_E)\\ &= \left\{-(1-K)H(f(\tau_E,K),\tau_E,\tau_i)f_{s_E}(f(\tau_E,K))\frac{\sqrt{(\tau_z+\tau_E)(1+\tau_z+\tau_E)}}{\tau_E\phi(\Phi^{-1}(K))}-\int_{f(\tau_E,K)}^{\infty} H(s_E,\tau_E,\tau_i)dF(s_E)\right\}(1-G_L(F-y_L+I_y))\\ &< 0\end{aligned}$$

Result: $H(s_E,\tau_E,\tau_i)$ increases with τ_i.

Proof: Recall that

$$H(s_E,\tau_E,\tau_i) = \int_{w_E}^{\infty}\{\Phi(w_i)-\Phi(w_E)\}\,dF_{w_i|s_E}(w_i) \tag{29}$$

$$\approx \phi(w_E)\int_{w_E}^{\infty}(w_i-w_E)\,dF_{w_i|s_E}(w_i) \tag{30}$$

$$= \phi(w_E)\left[\mu_i\Phi\left(\frac{\mu_i}{\sigma_i}\right)+\sigma_i\phi\left(\frac{\mu_i}{\sigma_i}\right)\right] \tag{31}$$

where $\mu_i=\sqrt{\frac{\psi_i}{1+\psi_i}}\frac{\tau_z\mu_z+\tau_E s_E}{\tau_z+\tau_E}-w_E, \sigma_i=\frac{\tau_i}{(1+\psi_i)(\tau_z+\tau_E)}$. Differentiating H wrt τ_i,

$$\frac{\partial H}{\partial \tau_i}=\phi(w_E)\left[\Phi\left(\frac{\mu_i}{\sigma_i}\right)\frac{\partial \mu_i}{\partial \tau_i}+\phi\left(\frac{\mu_i}{\sigma_i}\right)\frac{\partial \sigma_i}{\partial \tau_i}\right]$$

It is obvious that when both the mean and variance of distribution of $w_i|s_E$ increases with τ_i, H increases with τ_i as well. Next, we will show that this result holds more generally. Taking derivative of μ_i and σ_i with respect to τ_i, we get $\frac{\partial \mu_i}{\partial \tau_i}=\frac{\mu_i}{2\psi_i(1+\psi_i)}$ and $\frac{\partial \sigma_i}{\partial \tau_i}=\frac{\sigma_i(1+\tau_Z+\tau_E)}{2\tau_i(1+\psi_i)}$. So, H increases with τ_i if

$$\lambda\Phi(\lambda)+\phi(\lambda)(1+\frac{\tau_Z+\tau_E}{\tau_i})(1+\tau_Z+\tau_E)>0$$

where $\lambda = \frac{\mu_i}{\sigma_i}$. It is clear that

$$\lambda\Phi(\lambda) + \phi(\lambda) > 0 \qquad \forall\lambda > 0 \implies \frac{\partial H}{\partial \tau_i} > 0 \quad \forall\lambda > 0$$

The challenge is to show it for negative values of λ. Using Chebychev's inequality for standard normal random variable X, we know that

$$E[X|X > \lambda] > \lambda.$$

Note that lhs of the above expression can be simplified as $E[X|X > \lambda] = \frac{\phi(\lambda)}{\Phi(-\lambda)}$. Substituting this, we get

$$-\lambda\Phi(-\lambda) + \phi(-\lambda) > 0 \qquad \forall\lambda \implies \frac{\partial H}{\partial \tau_i} > 0 \quad \forall\lambda$$

In case 1, taking derivative of expected utility wrt y_H gives

$$\frac{\partial EU}{\partial y_H} = \left\{ KH(f(\tau_E,K),\tau_E,\tau_i) f_{s_E}(f(\tau_E,K)) \frac{\sqrt{(\tau_z+\tau_E)(1+\tau_z+\tau_E)}}{\tau_E \phi(\Phi^{-1}(K))} + \int_{f(\tau_E,K)}^{\infty} H(s_E,\tau_E,\tau_i) dF(s_E) \right\} (1 - G_H(F - y_H + I_y)).$$

The fact that H is monotonic in τ_i implies that marginal value of acquiring information increases with y_H. We can use similar logic to prove other statements in the theorem. ■

Proof of Proposition 3.4. Equation **??** has unique symmetric solution when $\frac{\partial EU(\tau_i,\tau_E)}{\partial \tau_i}|_{\tau_i=\tau_j=\tau \forall j} = \frac{\partial C(\tau_i)}{\partial \tau_i}|_{\tau_i=\tau}$. Since the cost function is convex, the rhs of above equation is increasing in τ. Equation **??** has unique solution when lhs is decreasing in τ. This is true when

$$\frac{\partial^2 EU(\tau_i,\tau_E)}{\partial \tau_i^2}|_{\tau_i=\tau_j=\tau} + \frac{\partial^2 EU}{\partial \tau_i \partial \tau_E}|_{\tau_i=\tau_j=\tau} < 0$$

This is true given the concavity of EU and when there is substitutability across investors. ■

Proof of Lemma 4.1. (1) In case 1, we can rewrite the condition for $NPV < 0$ as

$$\frac{\mathbb{E}[q]}{1-\mathbb{E}[q]} < \frac{I_y - y_L}{y_H - I_y}$$

Similarly, $NPV_E < 0$ if

$$\begin{aligned}
\frac{\mathbb{E}[q]}{1-\mathbb{E}[q]} &< \frac{E_L(F,\mathbf{0}) - E_L(F,\mathbf{c_L})}{E_H(F,\mathbf{c_H}) - E_H(F,\mathbf{0})} \\
&= \frac{\int_F^{\infty}(I_y - y_L)dG_L + \int_F^{F-(y_L-I_y)}[x-(F-(y_L-I_y))]dG_L}{\int_F^{\infty}(y_H - I_y)dG_H + \int_{F-(y_H-I_y)}^{F}[x-(F-(y_H-I_y))]dG_H} \\
&= \left[\frac{I_y - y_L}{y_H - I_y}\right]\left[\frac{1-G_L(F) + \frac{\int_F^{F-(y_L-I_y)}[x-(F-(y_L-I_y))]dG_L}{I_y - y_L}}{1-G_H(F) + \frac{\int_{F-(y_H-I_y)}^{F}[x-(F-(y_H-I_y))]dG_H}{y_H - I_y}}\right] \\
&< \frac{I_y - y_H}{y_L - I_y}
\end{aligned}$$

The last inequality holds because it is always the case that (1) $\int_F^{F-(y_L-I_y)}[x-(F-(y_L-I_y))]dG_L < 0$ and $\int_{F-(y_H-I_y)}^{F}[x-(F-(y_H-I_y))]dG_H > 0$, while (2) $G_H(F) < G_L(F)$ holds by assumption of FOSD without investment. By the same logic, it is straightforward to see that conditions exist under which $NVP_E > 0$, while $NPV < 0$.

(2) In case 2, we can rewrite the condition for $NPV > 0$ as

$$\frac{1-\mathbb{E}[q]}{\mathbb{E}[q]} > \frac{I_y - y_H}{y_L - I_y}$$

Similarly, $NPV_E > 0$ if

$$\begin{aligned}
\frac{1-\mathbb{E}[q]}{\mathbb{E}[q]} &> \frac{E_H(F,\mathbf{0}) - E_H(F,\mathbf{c_H})}{E_L(F,\mathbf{c_L}) - E_L(F,\mathbf{0})} \\
&= \frac{\int_{F-(y_H-I_y)}^{\infty}(I_y - y_H)dG_H + \int_F^{F-(y_H-I_y)}(x-F)dG_H}{\int_{F-(y_L-I_y)}^{\infty}(y_L - I_y)dG_L + \int_{F-(y_L-I_y)}^{F}(x-F)dG_L} \\
&= \left[\frac{I_y - y_H}{y_L - I_y}\right]\left[\frac{1-G_H(F-(y_H-I_y)) + \frac{\int_F^{F-(y_H-I_y)}(x-F)dG_H}{I_y - y_H}}{1-G_L(F-(y_L-I_y)) + \frac{\int_{F-(y_L-I_y)}^{F}(x-F)dG_L}{y_L - I_y}}\right] \\
&> \frac{I_y - y_H}{y_L - I_y}
\end{aligned}$$

The last inequality holds because it is always the case that (1) $\int_F^{F-(y_H-I_y)}(x-F)dG_H > 0$ and

$\int_{F-(y_L-I_y)}^{F}(x-F)dG_L < 0$, while (2) $G_H(F+I_y-y_H) < G_L(F+I_y-y_L)$ holds by assumption of FOSD with investment. By the same logic, it is straightforward to see that conditions exist under which $NVP_E < 0$, while $NPV > 0$. ■

Proof of Corollary 4.2. This follows directly from the lemma above. ■

Proof of Proposition 4.3. Recall that expected utility of acquiring information of precision τ_i when prices reveal information of precision τ_E is given by

$$EU(\tau_i,\tau_E) = \Delta E(F,0) \int_{-\infty}^{f(\tau_E,K)} H(s_E,\tau_E,\tau_i)dF_{s_E}(s_E) + \Delta E(F,c) \int_{f(\tau_E,K)}^{\infty} H(s_E,\tau_E,\tau_i)dF_{s_E}(s_E)$$

Taking partial derivative with respect to τ_E gives us

$$\frac{\partial EU}{\partial \tau_E} = \Delta E(F,0) \int_{-\infty}^{f(\tau_E,K)} \frac{\partial\left(H(s_E,\tau_E,\tau_i)f_{s_E}(s_E,\tau_E)\right)}{\partial \tau_E} ds_E + \Delta E(F,c) \int_{f(\tau_E,K)}^{\infty} \frac{\partial\left(H(s_E,\tau_E,\tau_i)f_{s_E}(s_E,\tau_E)\right)}{\partial \tau_E} ds_E - \quad (32)$$

$$(\Delta E(F,c)-\Delta E(F,0)) \frac{\partial f(\tau_E,K)}{\partial \tau_E} H(f(\tau_E,K),\tau_E,\tau_i) f_{s_E}(f(\tau_E,K),\tau_E) \quad (33)$$

Lets focus on the third term first. For the sake of simplicity, let $\mu_z = 0$. Using this, we can write

$$\frac{\partial f(\tau_E,K)}{\partial \tau_E} = \frac{\partial}{\partial \tau_E}\left(\frac{\Phi^{-1}(K)\left[\sqrt{(\tau_z+\tau_E)(1+\tau_z+\tau_E)}\right]}{\tau_E}\right) < 0 \iff K > 0.5 \iff \text{The project if -ve } NPV_E \quad (34)$$

This implies that if the project is negative NPV equity there could be complementarity.

Lets focus on case 2 now. In this case,

$$\frac{\partial EU}{\partial \tau_E} = \Delta E(F,c) \int_{-\infty}^{f(\tau_E,K)} \frac{\partial\left(H(s_E,\tau_E,\tau_i)f_{s_E}(s_E,\tau_E)\right)}{\partial \tau_E} ds_E + \Delta E(F,0) \int_{f(\tau_E,K)}^{\infty} \frac{\partial\left(H(s_E,\tau_E,\tau_i)f_{s_E}(s_E,\tau_E)\right)}{\partial \tau_E} ds_E + \quad (35)$$

$$(\Delta E(F,c)-\Delta E(F,0)) \frac{\partial f(\tau_E,K)}{\partial \tau_E} H(f(\tau_E,K),\tau_E,\tau_i) f_{s_E}(f(\tau_E,K),\tau_E) \quad (36)$$

Here again, lets focus on the third term first. We will have complementarity if the third term is positive. This is true if $\frac{\partial f(\tau_E,K)}{\partial \tau_E} < 0$ i.e., the project is positive NPV equity. ■

Proof of Lemma 4.4. Taking partial derivative of NPV wrt α, we get

$$\frac{\partial NPV}{\partial \alpha} = q_0 \frac{\partial c_H}{\partial \alpha} + (1-q_0)\frac{\partial c_L}{\partial \alpha} \tag{37}$$

$$= \left((1+\gamma)\frac{q_0}{1-G_H(F-c_H)} - \frac{1-q_0}{1-G_L(F-c_L)}\right) \tag{38}$$

$$= \left(\frac{q_0(1+\gamma)(1-G_L(F-c_L)) - (1-q_0)(1-G_H(F-c_H))}{(1-G_H(F-c_H))(1-G_L(F-c_L))}\right) \tag{39}$$

This is less than zero when

$$\gamma < \frac{(1-q_0)(1-G_H(F))}{q_0(1-G_L(F))} - 1 \equiv \bar{\gamma}$$

Moreover, for NPV_E to be positive, we need

$$\gamma > \frac{1}{q_0} - 2 \equiv \underline{\gamma}$$

■

Proof of Proposition 4.5. Taking partial derivative of expected utility with respect to α in case 1 gives us

$$\begin{aligned}
\frac{\partial EU}{\partial \alpha} &= (\Delta E(F,0) - \Delta E(F,c))H(f(\tau_E,K),\tau_E,\tau_i)f_{s_E}(f(\tau_E,K))\frac{\partial f(\tau_E,K)}{\partial \alpha} + \frac{\partial \Delta E(F,c)}{\partial \alpha}\int_{f(\tau_E,K)}^{\infty} H(s_E,\tau_E,\tau_i)dF_{s_E}(s_E) \\
&= (\Delta E(F,0) - \Delta E(F,c))H(f(\tau_E,K),\tau_E,\tau_i)f_{s_E}(f(\tau_E,K))\frac{\sqrt{(\tau_z+\tau_E)(1+\tau_z+\tau_E)}}{\tau_E\phi(\Phi^{-1}(K))}\frac{\partial K}{\partial \alpha} + \frac{1}{K_0}\int_{f(\tau_E,K)}^{\infty} H(s_E,\tau_E,\tau_i)dF_{s_E}(s_E) \\
&= \frac{1}{K_0}\int_{f(\tau_E,K)}^{\infty} H(s_E,\tau_E,\tau_i)dF_{s_E}(s_E) \\
&> 0
\end{aligned}$$

This implies that the marginal value of acquiring information increases with α. ■

Proof of Proposition 4.6. (i) Probability of inefficient investment is given by

$$\Phi\left(\underbrace{\frac{\Phi^{-1}(K_0)\sqrt{(1+\tau_z+\tau_E)\tau_z} - \mu_z\sqrt{(\tau_z+\tau_E)\tau_z}}{\sqrt{\tau_E}}}_{\equiv\varpi_1}\right) - \Phi\left(\underbrace{\frac{\Phi^{-1}(K)\sqrt{(1+\tau_z+\tau_E)\tau_z} - \mu_z\sqrt{(\tau_z+\tau_E)\tau_z}}{\sqrt{\tau_E}}}_{\equiv\varpi_2}\right)$$

Differentiating this probability wrt τ_E gives us

$$\propto \phi(\varpi_1)\left(-\frac{\Phi^{-1}(K_0)(1+\tau_z)}{\sqrt{1+\tau_z+\tau_E}}+\frac{\mu_z\tau_z}{\sqrt{\tau_z+\tau_E}}\right)-\phi(\varpi_2)\left(-\frac{\Phi^{-1}(K)(1+\tau_z)}{\sqrt{1+\tau_z+\tau_E}}+\frac{\mu_z\tau_z}{\sqrt{\tau_z+\tau_E}}\right) \tag{40}$$

$$=\frac{1+\tau_z}{\sqrt{1+\tau_z+\tau_E}}\left(\phi(\varpi_2)\Phi^{-1}(K)-\phi(\varpi_1)\Phi^{-1}(K_0)\right)+\frac{\mu_z\tau_z}{\sqrt{\tau_z+\tau_E}}(\phi(\varpi_1)-\phi(\varpi_2)) \tag{41}$$

Note that $K_0 > K$ implies that $\varpi_1 > \varpi_2$. We want the above expression (41) to be negative. First note that, condition $\gamma > \bar{\gamma}$ implies that $\varpi_2 < 0$.

If $\frac{\phi(\varpi_1)}{\phi(\varpi_2)} < 1$, the second term in equation 41 is negative. Moreover, if $\frac{\Phi^{-1}(K)}{\Phi^{-1}(K_0)} < \frac{\phi(\varpi_1)}{\phi(\varpi_2)}$, the first term in equation 41 is also negative.

(ii) Probability of inefficient investment conditional of investment taking place is

$$\frac{\Phi(\varpi_1)-\Phi(\varpi_2)}{1-\Phi(\varpi_2)}=\frac{\Phi(-\varpi_2)-\Phi(-\varpi_1)}{\Phi(-\varpi_2)}=1-\frac{\Phi(-\varpi_1)}{\Phi(-\varpi_2)}$$

Differentiating the above with respect to τ_E gives us

$$\propto \phi(\varpi_1)\left(-\frac{\Phi^{-1}(K_0)(1+\tau_z)}{\sqrt{1+\tau_z+\tau_E}}+\frac{\mu_z\tau_z}{\sqrt{\tau_z+\tau_E}}\right)-\phi(\varpi_2)\left(-\frac{\Phi^{-1}(K)(1+\tau_z)}{\sqrt{1+\tau_z+\tau_E}}+\frac{\mu_z\tau_z}{\sqrt{\tau_z+\tau_E}}\right)\frac{\Phi(-\varpi_1)}{\Phi(-\varpi_2)} \tag{42}$$

$$=\frac{1+\tau_z}{\sqrt{1+\tau_z+\tau_E}}\left(\frac{\Phi(-\varpi_1)}{\Phi(-\varpi_2)}\phi(\varpi_2)\Phi^{-1}(K)-\phi(\varpi_1)\Phi^{-1}(K_0)\right)+\frac{\mu_z\tau_z}{\sqrt{\tau_z+\tau_E}}\left(\phi(\varpi_1)-\phi(\varpi_2)\frac{\Phi(-\varpi_1)}{\Phi(-\varpi_2)}\right) \tag{43}$$

If $\frac{\Phi^{-1}(K)}{\Phi^{-1}(K_0)}\frac{\Phi(-\varpi_1)}{\Phi(-\varpi_2)} < \frac{\phi(\varpi_1)}{\phi(\varpi_2)} < \frac{\Phi(-\varpi_1)}{\Phi(-\varpi_2)}$, the conditional probability of inefficient investment decreases with more learning. So, the necessary condition for both to be true is $\frac{\Phi^{-1}(K)}{\Phi^{-1}(K_0)} < \frac{\phi(\varpi_1)}{\phi(\varpi_2)} < \frac{\Phi(-\varpi_1)}{\Phi(-\varpi_2)}$ ■

Proof of Lemma 4.7. Taking partial derivative of NPV wrt α, we get

$$
\begin{aligned}
\frac{\partial NPV}{\partial \alpha} &= q_0 \frac{\partial c_H}{\partial \alpha} + (1-q_0)\frac{\partial c_L}{\partial \alpha} && (44)\\
&= -\left((1+\gamma)\,\frac{q_0}{1-G_H(F-c_H)} - \frac{1-q_0}{1-G_L(F-c_L)}\right) && (45)\\
&= -\left(\frac{q_0(1+\gamma)(1-G_L(F-c_L)) - (1-q_0)(1-G_H(F-c_H))}{(1-G_H(F-c_H))(1-G_L(F-c_L))}\right) && (46)
\end{aligned}
$$

NPV increases with α whenever γ is small enough and $\alpha \in [0, \bar{\alpha}]$ ■

Proof of Proposition 4.8. Taking partial derivative of expected utility with respect to α in case 2 gives us

$$
\begin{aligned}
\frac{\partial EU}{\partial \alpha} &= (\Delta E(F,c) - \Delta E(F,0))H(f(\tau_E,K),\tau_E,\tau_i) f_{s_E}(f(\tau_E,K)) \frac{\partial f(\tau_E,K)}{\partial \alpha} + \frac{\partial \Delta E(F,c)}{\partial \alpha} \int_{-\infty}^{f(\tau_E,K)} H(s_E,\tau_E,\tau_i)\, dF_{s_E}(s_E)\\
&= (\Delta E(F,c) - \Delta E(F,0))H(f(\tau_E,K),\tau_E,\tau_i) f_{s_E}(f(\tau_E,K)) \frac{\sqrt{(\tau_z+\tau_E)(1+\tau_z+\tau_E)}}{\tau_E \phi(\Phi^{-1}(K))} \frac{\partial K}{\partial \alpha} - \frac{1}{K_0} \int_{-\infty}^{f(\tau_E,K)} H(s_E,\tau_E,\tau_i)\, dF_{s_E}(s_E)\\
&= -\frac{1}{K_0} \int_{-\infty}^{f(\tau_E,K)} H(s_E,\tau_E,\tau_i)\, dF_{s_E}(s_E)\\
&< 0
\end{aligned}
$$

The above expression is always negative which implies that, as project become more efficient, investors marginal benefit to acquire information decreases. ■

Proof of Proposition 4.9. (i) The probability of efficient investment not taken is

$$
\Phi\left(\underbrace{\frac{\Phi^{-1}(K)\sqrt{(1+\tau_z+\tau_E)\tau_z} - \mu_z\sqrt{(\tau_z+\tau_E)\tau_z}}{\sqrt{\tau_E}}}_{\equiv \varpi_1}\right) - \Phi\left(\underbrace{\frac{\Phi^{-1}(K_0)\sqrt{(1+\tau_z+\tau_E)\tau_z} - \mu_z\sqrt{(\tau_z+\tau_E)\tau_z}}{\sqrt{\tau_E}}}_{\equiv \varpi_2}\right)
$$

Differentiating this probability wrt τ_E gives us

$$
\begin{aligned}
&\propto \phi(\varpi_1)\left(-\frac{\Phi^{-1}(K)(1+\tau_z)}{\sqrt{1+\tau_z+\tau_E}} + \frac{\mu_z\tau_z}{\sqrt{\tau_z+\tau_E}}\right) - \phi(\varpi_2)\left(-\frac{\Phi^{-1}(K_0)(1+\tau_z)}{\sqrt{1+\tau_z+\tau_E}} + \frac{\mu_z\tau_z}{\sqrt{\tau_z+\tau_E}}\right) && (47)\\
&= \frac{1+\tau_z}{\sqrt{1+\tau_z+\tau_E}}\left(\phi(\varpi_2)\Phi^{-1}(K_0) - \phi(\varpi_1)\Phi^{-1}(K)\right) + \frac{\mu_z\tau_z}{\sqrt{\tau_z+\tau_E}}(\phi(\varpi_1) - \phi(\varpi_2)) && (48)
\end{aligned}
$$

Note that $K_0 < K$ implies that $\varpi_1 > \varpi_2$. We want the above expression (48) to be negative.

If $\frac{\phi(\varpi_1)}{\phi(\varpi_2)} < 1$, the second term in equation 48 is negative. Moreover, if $\frac{\Phi^{-1}(K)}{\Phi^{-1}(K_0)} < \frac{\phi(\varpi_1)}{\phi(\varpi_2)}$, the first term in equation 48 is also negative.

(ii) conditional probability of efficient investment not taken is given by

$$\frac{\Phi(\varpi_1) - \Phi(\varpi_2)}{\Phi(\varpi_1)} = 1 - \frac{\Phi(\varpi_2)}{\Phi(\varpi_1)}$$

Differentiating the above with respect to τ_E gives us

$$\propto \phi(\varpi_1)\left(-\frac{\Phi^{-1}(K)(1+\tau_z)}{\sqrt{1+\tau_z+\tau_E}} + \frac{\mu_z \tau_z}{\sqrt{\tau_z+\tau_E}}\right) - \phi(\varpi_2)\left(-\frac{\Phi^{-1}(K_0)(1+\tau_z)}{\sqrt{1+\tau_z+\tau_E}} + \frac{\mu_z \tau_z}{\sqrt{\tau_z+\tau_E}}\right)\frac{\Phi(\varpi_1)}{\Phi(\varpi_2)} \quad (49)$$

$$= \frac{1+\tau_z}{\sqrt{1+\tau_z+\tau_E}}\left(\frac{\Phi(\varpi_1)}{\Phi(\varpi_2)}\phi(\varpi_2)\Phi^{-1}(K_0) - \phi(\varpi_1)\Phi^{-1}(K)\right) + \frac{\mu_z \tau_z}{\sqrt{\tau_z+\tau_E}}\left(\phi(\varpi_1) - \phi(\varpi_2)\frac{\Phi(\varpi_1)}{\Phi(\varpi_2)}\right) \quad (50)$$

If $\frac{\Phi^{-1}(K_0)}{\Phi^{-1}(K)}\frac{\Phi(\varpi_1)}{\Phi(\varpi_2)} < \frac{\phi(\varpi_1)}{\phi(\varpi_2)} < \frac{\Phi(\varpi_1)}{\Phi(\varpi_2)}$, the conditional probability of inefficient investment decreases with more learning. ■

Proof of Proposition 5.1. Recall that expected utility of acquiring information of precision τ_i when prices reveal information of precision τ_E is given by

$$EU(\tau_i, \tau_E) = \Delta E(F,0) \int_{-\infty}^{f(\tau_E,K)} H(s_E,\tau_E,\tau_i) dF_{s_E}(s_E) + \Delta E(F,c) \int_{f(\tau_E,K)}^{\infty} H(s_E,\tau_E,\tau_i) dF_{s_E}(s_E)$$

Taking partial derivative with respect to τ_E gives us

$$\frac{\partial EU}{\partial \tau_E} = \Delta E(F,0) \int_{-\infty}^{f(\tau_E,K)} \frac{\partial\left(H(s_E,\tau_E,\tau_i) f_{s_E}(s_E,\tau_E)\right)}{\partial \tau_E} ds_E + \Delta E(F,c) \int_{f(\tau_E,K)}^{\infty} \frac{\partial\left(H(s_E,\tau_E,\tau_i) f_{s_E}(s_E,\tau_E)\right)}{\partial \tau_E} ds_E - \quad (51)$$

$$(\Delta E(F,c) - \Delta E(F,0)) \frac{\partial f(\tau_E,K)}{\partial \tau_E} H(f(\tau_E,K),\tau_E,\tau_i) f_{s_E}(f(\tau_E,K),\tau_E) \quad (52)$$

Lets focus on the third term first. For the sake of simplicity, let $\mu_z = 0$. Using this, we can write

$$\frac{\partial f(\tau_E,K)}{\partial \tau_E} = \frac{\partial}{\partial \tau_E}\left(\frac{\Phi^{-1}(K)\left[\sqrt{(\tau_z+\tau_E)(1+\tau_z+\tau_E)}\right]}{\tau_E}\right) < 0 \iff K > 0.5 \iff \text{The project if -ve } NPV_E \quad (53)$$

This implies that if the project is negative NPV equity, there could be complementarity.

Lets focus on case 2 now. In this case,

$$\frac{\partial EU}{\partial \tau_E} = \Delta E(F,c) \int_{-\infty}^{f(\tau_E,K)} \frac{\partial \left(H(s_E,\tau_E,\tau_i) f_{s_E}(s_E,\tau_E) \right)}{\partial \tau_E} ds_E + \Delta E(F,0) \int_{f(\tau_E,K)}^{\infty} \frac{\partial \left(H(s_E,\tau_E,\tau_i) f_{s_E}(s_E,\tau_E) \right)}{\partial \tau_E} ds_E + \tag{54}$$

$$(\Delta E(F,c) - \Delta E(F,0)) \frac{\partial f(\tau_E,K)}{\partial \tau_E} H(f(\tau_E,K),\tau_E,\tau_i) f_{s_E}(f(\tau_E,K),\tau_E) \tag{55}$$

Here again, lets focus on the third term first. We will have complementarity if the third term is positive. This is true if $\frac{\partial f(\tau_E,K)}{\partial \tau_E} < 0$ i.e., the project is positive NPV equity. ■